Increase in Capacity of Multiuser OFDM System Using Dynamic Subchannel Allocation

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***Abstract*—This paper investigates the problem of dynamic multiuser subchannel allocation in the downlink of OFDM systems. The assumptions are that the channel model is quasi-static and that the base station has perfect channel information. In traditional TDMA or FDMA systems, resource allocation for each user is non-adaptively fixed, and the water-filling power spectrum is known to be optimal. Since the subchannel allocations among the users are not optimized, a group of users is likely to suffer from poor channel gains resulting from large path loss and random fading. To resolve this problem, we derive a multiuser convex optimization problem to find the optimal allocation of subchannels, and propose a low-complexity adaptive subchannel allocation algorithm. Simulation results show that the proposed algorithm performs almost as well as the optimal solution. Also, higher spectral efficiency is achieved for larger number of users in a cell due to the multiuser diversity.

I. INTRODUCTION

Successful deployment of wireless voice communication systems promises a bright future for wireless high data rate services such as internet access or multimedia applications. To provide such high data rate services, OFDM (Orthogonal Frequency Division Multiplexing) is being considered as a viable candidate [1] [2] due to its ability to overcome multipath fading.

Achieving the capacity of the multicarrier systems such as OFDM has been well-studied. If the channel is static and is perfectly known to the transmitter and the receiver, water-filling [3] with adaptive modulation [4] is known to be optimal. Water-filling also applies if the channel is slowly fading as in fixed wireless systems or if the channel estimation and feedback can be performed in a short time span. Note that faster channel estimation is increasingly possible as system bandwidth becomes larger. Although adaptive loading has been hardly included in actual implementation due to the time varying nature of wireless channels, it is starting to receive more attention as the spectral efficiency becomes more important and complexity less an issue.

However, the water-filling solution can only be used for single-user systems or multiuser systems with fixed resource assignment. For example, in TDMA (Time Division Multiple Access) or FDMA (Frequency Division Multiple Access) systems with non-adaptive fixed resource allocation, an independent dimension such as time slot or frequency band is assigned to each user regardless of channel responses. In this case, each user effectively becomes a single user who is independent of all other users. Then, water-filling solution can be used for each user to maximize its throughput. However, the maximized rate is far below the rate that can be achieved by adaptive resource allocation, and a set of users suffers from poor channel gains of assigned dimensions. Larger throughput can be achieved if we assign resources adaptively, i.e. if we assign a dimension to users whose channel gains are good for it.

The characterization of information theoretic channel capacity for a multiuser system is a complex optimization problem. To achieve channel capacity, highly complex coding and decoding such as maximum likelihood detection or multiuser detection with successive decoding need to be used [3]. Therefore, we restrict our focus on exclusive assignment of each resource dimension to only one user to avoid the complexity and the error propagation problems. In other words, we allow only one user to occupy a dimension related to a specific frequency at a specific time.

In this paper, we assume frequency selective quasi-static channels where channels do not vary within a block of transmission. Zero delay constraint, total power constraints, and independent uniform distribution of user location are assumed. Larger capacity can be found for relaxed delay constraint as more dimensions can be adaptively assigned. We show an optimal and propose a

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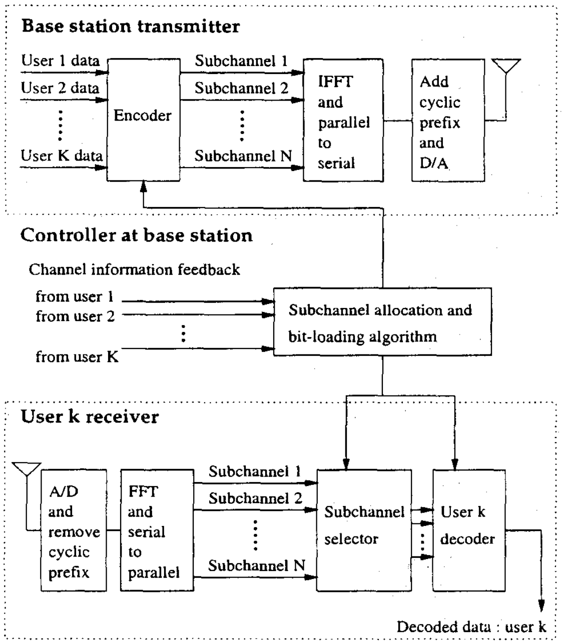

Fig. 1. Multiuser OFDM transmitter and receiver

suboptimal subchannel allocation algorithm for a multiuser OFDM downlink. Similar work can be done for uplink, but different power constraints need to be considered. Although only one total power constraint at base station exists for downlink, multiple total power constraints exist for uplink depending on number of users. Since we assume static channel within a block of transmission, only frequency dimensions need to be assigned adaptively. By adaptively assigning frequency subchannels, we can take advantage of channel diversity among users in different locations, which we call *Multiuser diversity*. This multiuser diversity stems from channel diversity including independent path loss and fading of users.

In Section II, we describe the system model and explain the objective. In Section III, we show how to find an optimal solution by formulating the given problem in a convex optimization problem. In Section IV, we propose a subchannel allocation algorithm and explain why its performance degradation from the optimal solution is expected to be small. In section V, we give simulation results of the proposed algorithm applied to various wireless channels.

II. SYSTEM MODEL AND OBJECTIVE

The structure of the multiuser OFDM system under consideration is shown in Figure 1. The controller at base station receives downlink channel information from all users, and controls base station transmitter using "subchannel allocation and bit-loading algorithm". K users in a hexagonal cell, total bandwidth B, and total number of subchannel N are assumed resulting in an OFDM subchannel bandwidth of $\frac{B}{N}$. The total bandwidth B need to be shared by K users. Maximum allowable total power for all users is P_{max}. Also, users are assumed to have an uniform distribution of location within the hexagonal cell. We assume path loss exponent of four. For each user, channel is assumed to be frequency selective Rayleigh fading, but similar results are obtained for frequency selective Ricean fading.

In a fixed TDMA scheme where time slots are nonadaptively assigned, users who have good channel responses can reliably receive higher data rate while others suffer from poor channel responses. With large path loss, this discrepancy cannot be ignored, and the system becomes unfair for the users with poor channel gains. In this paper, we assume VBR (Variable Bit Rate) services for all users, and maximize the smallest capacity of all users. Note that we mean capacity of a user by the maximum error-free data throughput with a proper coding for the given assignment of subchannels to the user. For different priority among users, or for different traffic models such as mixture of CBR (Constant Bit Rate) [5] and VBR, only a slight change on the algorithm is needed. A dual problem of minimizing total transmit power for given data rate requirements has been solved in [6].

III. OPTIMAL SOLUTION

The given problem can be formulated as

$$\begin{aligned}
\max_{P_{k,n},S_k}\ \min_{k} \quad & \sum_{n\subset S_k} \frac{B}{N} log_2 \left(1+\frac{P_{k,n}{h_{k,n}}^2}{N_0\frac{B}{N}}\right) \\
\text{subject to} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P_{max} \qquad (1)\\
& P_{k,n} \ge 0 \text{ for all } k,n \\
& S_1, S_2, \cdots S_K \text{ are disjoint} \\
& S_1 \cup S_2 \cup \cdots \cup S_K \subset \{1, 2 \cdots K\}
\end{aligned}$$

where $P_{k,n}$ is the power assigned to user k's subchannel n, $h_{k,n}$ is the channel gain of user k's subchannel n, S_k is the set of indices of subchannels assigned to user k, and N_0 is the power of additive white Gaussian noise (AWGN). $S_1, S_2, \cdots S_K$ need to be disjoint since we want to assign each dimension to one user and one user only. In this problem, we need to find optimal S_k and $P_{k,n}$ to maximize the minimum of all user's throughput. As set selection is involved with equation (1), it is not convex problem. However, we can convert this problem into a convex optimization problem by adopting a new parameter $\omega_{k,n}$, which represents portion of subchannel n assigned to user k.

$$\begin{aligned}
\max_{P_{k,n},\omega_{k,n}} \min_{k} \quad & \sum_{n=1}^{N} \frac{\omega_{k,n}B}{N} log_2 \left(1+\frac{P_{k,n}{h_{k,n}}^2}{N_0\frac{\omega_{k,n}B}{N}}\right) \\
\text{subject to} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P_{max} \\
& P_{k,n} \ge 0 \text{ for all } k,n \qquad (2)\\
& \sum_{k=1}^{K} \omega_{k,n} \le 1 \text{ for all } n \\
& \omega_{k,n} \ge 0 \text{ for all } k,n
\end{aligned}$$

Solving equation (2) gives an optimal solution with possibility of each subchannel being shared by users in FDMA manner. However, only a small number of subchannels are shared as $\omega_{k,n}$ is mostly either zero or one for $K \ll N$. [7] contains details of explanation.

Finally, by introducing a new variable t, the original problem can be formulated into a standard convex optimization problem which can be readily solved by standard packages such as AMPL [8].

$$\begin{aligned}
& \max_{P_{k,n},\omega_{k,n}} t, \text{ subject to} \\
& t \le \sum_{n=1}^{N} \frac{\omega_{k,n}B}{N} log_2 \left(1+\frac{P_{k,n}{h_{k,n}}^2}{N_0\frac{\omega_{k,n}B}{N}}\right) \text{ for all } k \\
& \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P_{max} \\
& P_{k,n} \ge 0 \text{ for all } k,n \qquad (3)\\
& \sum_{k=1}^{K} \omega_{k,n} \le 1 \text{ for all } n \\
& \omega_{k,n} \ge 0 \text{ for all } k,n
\end{aligned}$$

Solving this convex optimization problem gives the optimal solution for the max-min problem. However, this algorithm requires an intensive computation due to the recursive nature of solving a convex optimization problem. In simulation, equation (3) has been used to find the spectral efficiency of a multiuser OFDM system.

If we have mixture of VBR and CBR traffics, we can slightly change the formulation of equations to solve the problem. In this case, the problem is to maximize the minimum of VBR traffic throughputs while satisfying CBR traffic constraints. If users in set $S1$ require VBR traffics, and users in set $S2$ require CBR traffics, the new problem can be formulated as follow. R_k is the data rate required by a CBR traffic user k.

$$\begin{aligned}
& \max_{P_{k,n},\omega_{k,n}} t, \text{ subject to} \\
& t \le \sum_{n=1}^{N} \frac{\omega_{k,n}B}{N} log_2 \left(1+\frac{P_{k,n}{h_{k,n}}^2}{N_0\frac{\omega_{k,n}B}{N}}\right) \text{ for all } k \subset S1 \\
& \sum_{n=1}^{N} \frac{\omega_{k,n}B}{N} log_2 \left(1+\frac{P_{k,n}{h_{k,n}}^2}{N_0\frac{\omega_{k,n}B}{N}}\right) \ge R_k \text{ for all } k \subset S2 \\
& \sum_{n=1}^{N}\sum_{k=1}^{K} P_{k,n} \le P_{max} \\
& P_{k,n} \ge 0 \text{ for all } k,n \\
& \sum_{k=1}^{K} \omega_{k,n} \le 1 \text{ for all } n \\
& \omega_{k,n} \ge 0 \text{ for all } k,n
\end{aligned}$$

If priority exists among VBR users, scaling factor can be added to equation (3).

IV. SUBOPTIMAL SOLUTION

In single user water-filling solution, it is known that the total data throughput of a zero-margin system is close to capacity even with flat transmit power spectral density (PSD) as long as the energy is poured only into subchannels with good channel gains [9]. Also, in the optimal solution in Section III, each subchannel is assigned to a user whose channel gain is good for it. This implies that a flat transmit PSD would hardly reduce the data throughput of a multiuser OFDM system. Furthermore, flat PSD might be necessary if power mask constraint is tighter than the total power constraint. In the following suboptimal solution, equal amount of power ($= \frac{P_{max}}{N}$) is allocated to each subchannel, and the algorithm below is used to assign subchannels to users. $C(h_{k,n})$ is defined as

$$C(h_{k,n}) = \frac{B}{N} log_2 \left(1+\frac{\frac{P_{max}}{N}{h_{k,n}}^2}{N_0\frac{B}{N}}\right)$$

and R_k represents zero-margin data rate of user k for the given assignment of subchannels.

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1. Initialization
   set $R_k = 0$ for all $k = 1 \sim K$, A = {1, 2, 3, ··· K}
2. for $k$ = 1 to $K$ {
   (a) find n satisfying $| h_{k,n} | \geq | h_{k,j} |$ for all $j \in A$
   (b) update $R_k$ and A with the n from (a) :
       $R_k = C(h_{k,n})$, A = A - {n}
   }
3. while A $\neq \emptyset$ {
   (a) find k satisfying $R_k \leq R_i$ for all i, $0 \leq i \leq K$
   (b) for the found $k$, find n satisfying $| h_{k,n} | \geq | h_{k,j} |$ for all $j \in A$
   (c) update $R_k$ and A with the k and n :
       $R_k = R_k + C(h_{k,n})$, A = A - {n}
   }
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Even though channel swapping after subchannel assignment can be used to achieve capacity, it aws not utilized in simulation as the above algorithm already achieves results close to optimal solution. Complexity of this algorithm is almost negligible compared with the complexity of finding optimal allocation by solving equation (3). For mixture of VBR and CBR traffics, or for priority constraints among VBR traffics, step 3.(a) can be modified depending on objective.

V. SIMULATION RESULTS

In this section, we will present simulation results of our proposed algorithm applied to varuious system parameters. We have two parts of simulation results. The first part shows a performace comparison between the optimal and the suboptimal algorithms. The second part shows percentage capacity gain of the suboptimal algorithm over fixed TDMA resource allocation as a function of number of users. For both parts, maximum difference of path loss is assumed to be 40dB.

A. Comparison of Optimal and Suboptimal Algorithms

$B = 1$ MHz, $N = 40$, $K = 3$ and 1 μsec of RMS delay spread have been selected. We have chosen relatively small size of parameters to reduce simulation time of finding the optimal solution. Performace gap is expected to be smaller for larger N as the suboptimal algorithm approaches to the optimal one for $K \ll N$. Figure. 2 shows the spectral efficiency for WSNR ranging from 0 dB to 25dB. Here, WSNR is defined as the worst possible average SNR of a user on the boundary of hexagonal cell. The spectral efficiency loss is 2.5 ~ 4% for WSNR around 10dB and 1.5 ~ 2% for WSNR around 20dB. Also, note that the proposed algorithm performs even better than the mean capacity of fixed TDMA resource allocation schemes for WSNR ≥ 5dB.

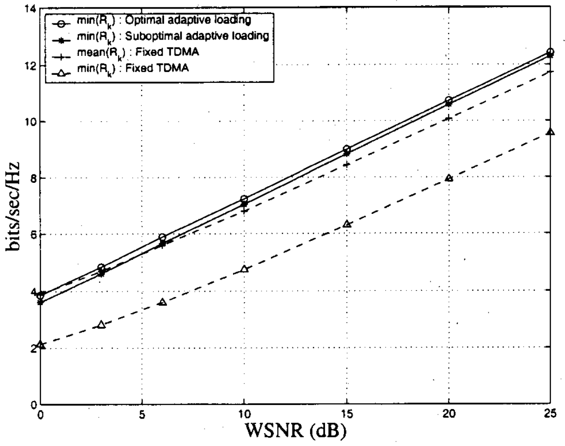

Fig. 2. *Spectral efficiency of optimal and suboptimal algorithms* Results for fixed TDMA resource allocation were shown for comparison. Three users in a cell was assumed.

B. Percentage Capacity Gain

We have selected $B = 10$ MHz, N = 512, $K = 1 \sim 20$, WSNR = 10dB and 0.1, 0.3, 1 and 5 μsec RMS delay spread for frequency selective channels. From Figure. 3, small difference in performance is observed for different delay spread as the path loss difference already gives enough multiuser diversty. Also, capacity gain is observed to be larger for larger number of users. If more users exist in a cell, the probability of at least one user having a very poor channel response increases, and adaptive subchannel allocation becomes more essential.

VI. CONCLUSIONS

This paper described an optimal subchannel allocation algorithm and proposed a suboptimal adaptive subchannel allocation algorithm for the downlink of an OFDM broadband system. Simulation results indicated that the proposed suboptimal algorithm with flat enerygy distribution over all subchannels can perform almost as well as the optimal power and subchannel allocation

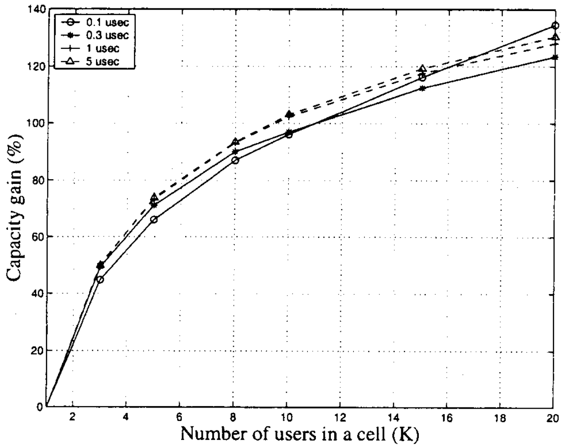

Fig. 3. *Percentage capacity gain VS number of users.* WSNR = 10dB, RMS delay spread = 0.1, 0.3, 1 and 5 μsec

scheme. This suboptimal algorithm offers a significant computational advantage while incurring small performance degradation. For the user with the worst downlink channel response, simulation results showed a 50~130% of capacity gain over a non-adaptive TDMA resource allocation scheme.

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