Analysis of a Simple Successive Interference Cancellation Scheme in a DS/CDMA System

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Abstract— **Compensating for near/far effects is critical for satisfactory performance of DS/CDMA systems. So far, practical systems have used power control to overcome fading and near/far effects. Another approach, which has a fundamental potential in not only eliminating near/far effects but also in substantially raising the capacity, is multiuser detection and interference cancellation. Various optimal and suboptimal schemes have been investigated. Most of these schemes, however, get too complex even for relatively simple systems and rely on good channel estimates. For interference cancellation, estimation of channel parameters (viz. received amplitude and phase) is important. We analyze a simple successive interference cancellation scheme for coherent BPSK modulation, where the parameter estimation is done using the output of a linear correlator. We then extend the analysis for a noncoherent modulation scheme, namely M-ary orthogonal modulation. For the noncoherent case, the needed information on both the amplitude and phase is obtained from the correlator output. Performance of the IC scheme along with multipath diversity combining is studied.**

I. Introduction

RECENTLY, there has been considerable interest in application of DS/CDMA (Direct Sequence Code Division Multiple Access) in cellular and personal communications. The choice of CDMA is attractive because of its potential capacity increases and other technical factors such as antimultipath fading capabilities.

Compensating for the near/far effect is critical for satisfactory performance of DS/CDMA systems. Commercial digital cellular system based on CDMA uses stringent power control, described in [1], to combat near/far effects and fading. Another approach, still in the research stage, is multiuser detection. In addition to mitigating the near/far effect, multiuser detection has the more fundamental potential of significantly raising capacity by cancelling multiple access interference. Valuable fundamental investigations (e.g., [2], [3]) have demonstrated huge potential capacity and performance improvements and have also shown the complexity of optimal structures. This has motivated the search for practical schemes achieving part of this potential. More information on multiuser detectors and interference cancellers can be found in a number of references, e.g., [4]–[8].

Manuscript received June 4, 1993; revised December 11, 1993. This paper was presented in part at the Fourth WINLAB Workshop on Third Generation Wireless Information Networks, October 19–10, 1993, East Brunswick, NJ USA.

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IEEE Log Number 9215588.

In general, a major problem with multiuser detectors and interference cancellers is the maintenance of simplicity. Even the suboptimal linear detectors have considerably complex processing, especially in an asynchronous channel. Certain schemes, where the users' signals are detected collectively, turn out to have a complex parallel structure. An alternative to parallel cancellation is to perform successive cancellation. The serial (successive) structure is not only more simple (requires less hardware), but is also more robust in doing the cancellations (see [7]). Comparison of the performance of the parallel and successive IC schemes is done in [9]. Successive cancellation was discussed in [10]. In [11] and [12], the CDMA-IC scheme involves successive cancellation and for this it relies on a gain list of users with nonincreasing strength. Our approach also successively cancels strongest users but assumes no knowledge of the users' powers. It uses the outputs of conventional correlation receiver to rank the users instead of separate channel estimates as in [11], [12][1]. Our approach [13], [9] is closer to that of [6], which uses the despread signals to select the strongest one for cancellation (but we cancel more than just the strongest). The approach in [14] also uses the information in the despread signal for cancellation.

We analyze the IC scheme operating by successively cancelling user interferences, ranked in order of received powers with the ranking obtained from correlations of the received signal with each user's chip sequence. In [5], the *bit decisions* are successively fed back in order of decreasing signal strength to improve performance. This occurs after matched filters and a decorrelating filter (matrix inversion). We do not use a decorrelator but rather successively feed back chip sequences based on decreasing signal strengths.

To perform interference cancellation, estimation of various parameter (viz. amplitude and phase) of the signals is important. For the coherent case, we shall assume the perfect knowledge of the phase is available at the receiver; however, amplitude estimation is required, which is accomplished by using the output of the linear correlator. In the noncoherent case, the phase knowledge is not required for the demodulation, but interference cancellation requires the knowledge of both the random phase and the amplitude. This knowledge of the amplitude and phase is also extracted from the output of the linear correlator.

The paper is organized as follows: Analysis of the IC scheme for BPSK modulation is done in Section II, followed by some results in Section III. In Section III, we also analyze

[1] The cancellation in [11], [12] is actually done in the spectral domain.

the performance of the IC scheme, under flat Rayleigh fading, using order statistics and the effect of averaging the correlations on the performance of the IC scheme. In Section IV, we extend the analysis of the IC scheme to a more robust modulation scheme, namely, M-ary orthogonal modulation along with multipath combining. Concluding comments are presented in Section V.

II. Successive IC Scheme for Coherent BPSK Modulation

We consider here a simple system to obtain basic results, where the received signal is

$$r(t) = \sum_{k=1}^{K} A_k \cdot a_k(t-\tau_k) \cdot b_k(t-\tau_k) \cdot \cos(\omega_c t + \phi_k) + n(t) \quad (1)$$

$r(t)$ = received signal
K = total number of active users
A_k = amplitude of k^{th} user
$b_k(t)$ = bit sequence of k^{th} user at bit-rate R_b
$a_k(t)$ = spreading chip sequence of k^{th} user at chip-rate R_c
$n(t)$ = additive white Gaussian noise (two sided power spectral density = $N_0/2$)
N = T/T_c where T = bit period and T_c = chip period

τ_k and ϕ_k are the time delay and phase of the k^{th} user, which are assumed to be known, i.e., tracked accurately.

The bits and chips are rectangular. Their values are all i.i.d. random values with probability 0.5 of ± 1. The τ_k and ϕ_k are i.i.d. uniform random variables in $[0, T]$ and $[0, 2\pi]$, respectively, for the asynchronous case.

We assume knowledge of the spread sequences of all the users, but no knowledge of the energies of the individual users is needed. As shown in the block diagram in Fig. 1, the basic idea is decoding the strongest users (whichever they may be), and then cancelling effects from the received signal. Here, the strongest user is not known beforehand, but is detected from the strength of the correlations of each of the users' chip sequence with the received signal. The correlation values (obtained from the conventional bank of correlators) are passed on to a selector which determines the strongest correlation value and selects the corresponding user for decoding and cancellation. These correlation values (as opposed to separate power estimates) form the basis for not only estimating the amplitude but also for maintaining the order of cancellation[2]. The process is repeated until the weakest user is decoded[3]. The flow chart of the process is shown in Fig. 2. Detailed analysis of the IC scheme for coherent BPSK system (after lowpass filtering) can be found in [13]. The results of the analysis are as follows.

After j cancellations, the decision variable for the $(j+1)^{\text{st}}$ user is given by

$$\widehat{Z_{j+1}} = \frac{1}{2} A_{j+1} b_{j+1} + \frac{1}{2} C_{j+1} \quad (2)$$

[2] Using the output of the correlator also helps in obtaining the knowledge of the phase during noncoherent demodulation (discussed later in Section IV).

[3] An alternative is to limit the number of cancellations. The impact of limiting the number of cancellations on the performance of the IC scheme is demonstrated in Section IV.

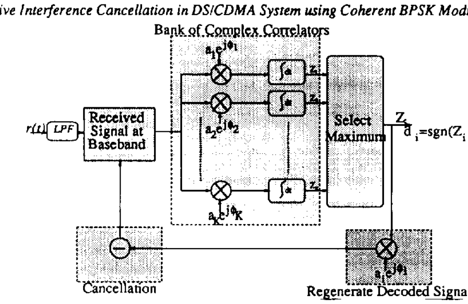

Fig. 1. Successive IC scheme with coherent BPSK modulation.

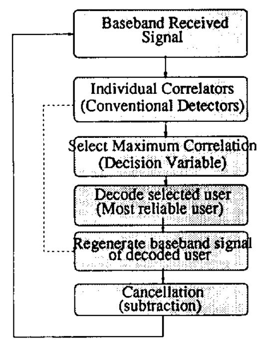

Fig. 2. Flow diagram of interference cancellation schemes.

and C_{j+1} is given by

$$C_{j+1} = \sum_{k=j+2}^{K} A_k I_{k,j+1}(\tau_{k,j+1}, \phi_{k,j+1}) + \left(n_{j+1}^I + n_{j+1}^Q\right) - \sum_{i=1}^{j} C_i I_{i,i+1}(\tau_{i,i+1}, \phi_{i,i+1}) \quad (3)$$

In the above expression, the first term is the multiple access interference of the uncancelled users, the second term is due to the Gaussian noise, and the third term is the cumulative noise due to imperfect cancellation. The cross correlation term is given by

$$I_{k,i}(\tau_{k,i}, \phi_{k,i}) = \frac{1}{T}\left[\int_0^T a_k(t-\tau_{k,i}) \cdot a_i(t) dt\right] \times \cos(\phi_k - \phi_i) \quad (4)$$

The variance of C_{j+1} conditioned on A_k as follows

$$\begin{aligned}\eta_{j+1} &= \text{Var}[C_{j+1} \mid A_k] \\ &= \sum_{k=j+2}^{K} A_k^2 \cdot \text{Var}[I_{k,j+1}(\tau_{k,j+1}, \phi_{k,j+1})] \\ &\quad + \text{Var}\left[\left(n_{j+1}^I + n_{j+1}^Q\right)\right] \\ &\quad + \sum_{i=1}^{j} \eta_i \cdot \text{Var}[I_{i,i+1}(\tau_{i,i+1}, \phi_{i,i+1})]\end{aligned} \quad (5)$$

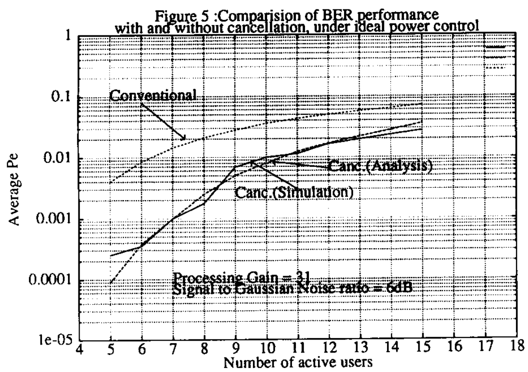

Fig. 3. Comparison of BER performance under ideal power control (synchronous case).

Further analysis yields

$$\eta_{j+1} = \begin{cases} \frac{1}{N}\sum_{k=j+2}^{K} A_k^2 + \frac{N_0}{T} + \frac{1}{N}\sum_{i=1}^{j}\eta_i & \text{; for Synchronous} \\ \frac{1}{3N}\sum_{k=j+2}^{K} A_k^2 + \frac{N_0}{T} + \frac{1}{3N}\sum_{i=1}^{j}\eta_i; & \text{for Asynchronous} \end{cases} \qquad (6)$$

The signal-to-noise ratio, conditioned on A_k, is then given by

$$\gamma_{j+1} = \frac{\frac{1}{4}A_{j+1}^2}{\frac{1}{4}\eta_{j+1}} = \frac{A_{j+1}^2}{\frac{1}{3N}\sum_{k=j+2}^{K} A_k^2 + \frac{N_0}{T} + \frac{1}{3N}\sum_{i=1}^{j}\eta_i}; \quad \text{for Asynchronous Case} \qquad (7)$$

To calculate the bit error rate, we shall use the Gaussian approximation [15], [16], i.e., we shall assume that the noise C_{j+1} is Gaussian with zero mean and variance η_{j+1}. The probability of bit error after the j^{th} cancellation, conditioned on the amplitudes A_k, is then given by

$$P_e^{j+1} = P\left\{\widehat{Z_{j+1}} < 0 \mid b_{j+1} = +1\right\} = P\{C_{j+1} < -A_{j+1}\} = Q\left(\frac{A_{j+1}}{\sqrt{\eta_{j+1}}}\right) = Q(\sqrt{\gamma_{j=1}}) \qquad (8)$$

Remark: Throughout the paper, approximations will be validated with simulations. The analysis assumes the users are cancelled in order of powers. The actual algorithm, using sample correlations to order the powers, may actually have this order changed. As will be seen, however, comparisons with simulation show good accuracy.

III. Results for Coherent BPSK

A. Performance of the IC Scheme Under Ideal Power Control

In Fig. 3, we compare the analysis and simulation results for users under ideal power control. The P_e's from analysis obtained using the Gaussian approximation agree well with the simulation. It is clear that the cancellation scheme works better than the conventional scheme even under ideal power control. Also, note that for similar P_e performance (0.01), six users in the conventional scheme can be increased to 11 users with cancellation, giving a substantial increase in capacity.

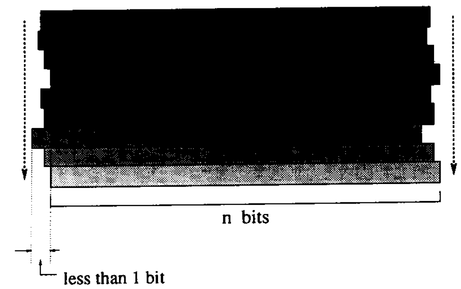

Fig. 4. Successive decoding in asynchronous channel.

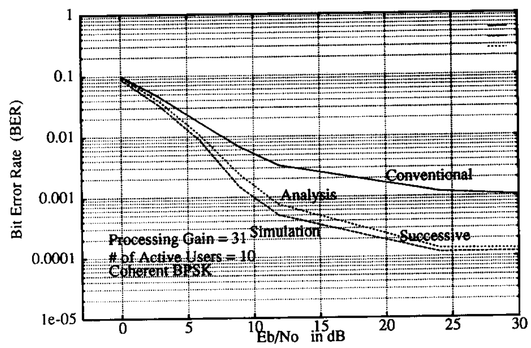

Fig. 5. BER versus E_b/N_o under ideal power control (asynchronous case).

Note on Asynchronous Channel: Introduction of asynchronism does not change the cancellation algorithm where the decoding and cancellation is done in decreasing order of the received signal strength. The only requirements (for coherent BPSK) of the cancellation algorithm are the timing and the phase knowledge (which are the same requirements for the conventional receiver). For asynchronous systems however, we must define what bits are compared with what other bits, and we propose the following. Group n bits of each user into a cancellation frame, where the maximum time between the first bit start and the last bit end is $(n + 1)$ bit times; see Fig. 4. After an entire frame is received, the correlations of the n bits of each user are averaged and the ranking of the users is obtained from these averages of correlations over n bits. This is what we used in our asynchronous simulations.[4] Issues in practical implementation are discussed in Section IV.

Fig. 5 is the BER versus the bit energy to noise ratio plots for the asynchronous case and under ideal power control. Total of 10 active users are present and the processing gain is 31.

Analysis of the cancellation scheme under fading is done next, where the order of cancellation changes as fast as the power level of the users change. More information on multiuser detectors in fading channels can be found in [17]–[19].

[4] For the results presented in this section, the ranking was obtained from averaging but the amplitude of the bit was estimated using correlation over a single bit. This was done for the purpose of controlling the simulation run times. In the sequel, we shall realize the improvement in performance by using the average of correlations over n bits for estimating the amplitude.

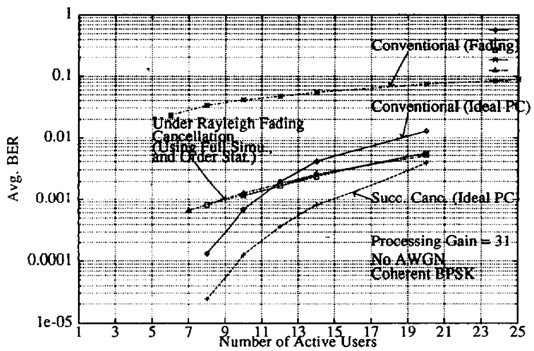

Fig. 6. Average BER versus number of users under Rayleigh fading.

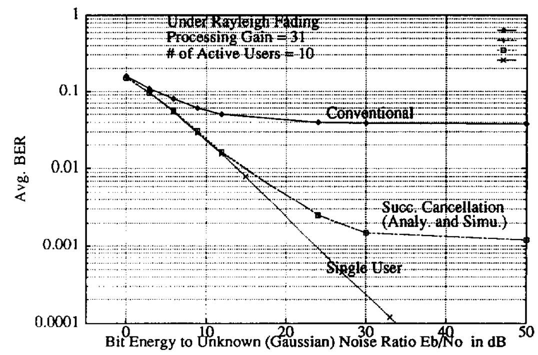

Fig. 7. Average BER versus E_b/N_o under Rayleigh fading.

B. Performance Analysis of the IC Scheme under Fading

Analysis Using Order Statistics: Analysis of the BER performance of the IC scheme under fading was done using order statistics [20]. Equations (6)-(8) are the expressions of the noise, signal-to-noise ratio, and the error probability resulting after j cancellations. These expressions are conditioned on A_k, which are the ordered set of amplitudes of K users. The amplitudes are assumed to be Rayleigh distributed with unit mean square value, i.e., its pdf is given by

$$f(x) = 2xe^{-x^2} \tag{9}$$

and its cdf (cumulative density function) is given by:

$$F(x) = 1 - e^{-x^2} \tag{10}$$

The pdf's of the ordered A_k (where A_1 is the strongest and A_K is the weakest) is denoted by $f_{A_k}(x)$ and is obtained as follows

$$f_{A_k}(x) = \frac{K!}{(K-k)!(k-1)!} F^{K-k}(x)[1-F(x)]^{k-1} f(x) \tag{11}$$

The expected values of the ordered A_k are then obtained as $E[A_k^2] = \int_0^\infty x_2 f_{A_k}(x)dx$. The denominator of (7) is approximated as Gaussian because the dominating terms for both small and large j are sums of random variables (all approximations will be validated by simulation). We have that (12) below. The error probability expression after the j^{th} cancellation (8) is then unconditioned using the pdf of the $j+1^{\text{th}}$ strongest amplitude as follows:

$$\widehat{P_e^{j+1}} = \int_0^\infty Q\left(\frac{A_{j+1}}{\sqrt{E_{A_k}[\eta_{j+1}]}}\right) f_{A_{j+1}}(x)dx \tag{13}$$

The average probability of error is then obtained as the average of the BER resulting from all stages of cancellation and is plotted in Fig. 6.

Comparison with Simulation Results: Monte-Carlo type simulations were run for the asynchronous case. The agreement of the actual computer simulation results with the analysis using order statistics is evident from Fig. 6.

From Fig. 6, it can also be seen that the performance of the successive IC scheme is much better compared to the conventional receiver under Rayleigh fading. Note that no coding is taken into consideration. So, the BER could be further lowered by using efficient coding. It should also be noted that Rayleigh fading is a worst case for two reasons. First of all, wideband CDMA transmission mitigates the Rayleigh fading effect [21]. Second, with CDMA, a Rake receiver can resolve the multipaths and improve performance. Interference cancellation with multipath resolution is analyzed later. The performance of the successive IC scheme under Rayleigh fading is comparable to that of the conventional receiver under ideal power control. This makes the successive IC scheme fading resistant.

Fig. 7 is the plot of the BER versus the bit energy to Gaussian noise (not including multiple access interference) ratio. Though the successive IC is much better than the conventional receiver under fading, its performance is not comparable to the single user detector (optimal detector). Further improvement in the IC scheme (by using averaging, described next) would, however, make it possible to achieve the single user bound.

C. Performance as a Function of Power Unbalance

The question as to the performance of the IC scheme as a function of power unbalance actually addresses two questions:

1) How does the IC scheme perform as the power unbalance increases?
2) How does the IC improvement over the conventional receiver change as the power unbalance increases?

Both of these questions are addressed in Fig. 8[5]. Since the IC scheme is a nonlinear scheme, it is difficult to draw a gen-

[5] Although Fig. 8 could be obtained from order statistic analysis like that of Section III, it was actually generated from a simulation. This explains the lack of smoothness.

$$E_{A_k}[\eta_{j+1}] = \begin{cases} \frac{1}{N}\sum_{k=j+2}^{K} E[A_k^2] + \frac{N_0}{T} + \frac{1}{N}\sum_{i=1}^{j}\eta_i; & \text{for Synchronous} \\ \frac{1}{3N}\sum_{k=j+2}^{K} E[A_k^2] + \frac{N_0}{T} + \frac{1}{3N}\sum_{i=1}^{j}\eta_i; & \text{for Asynchronous} \end{cases} \tag{12}$$

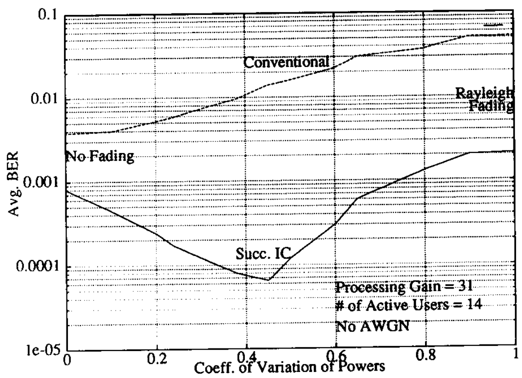

Fig. 8. Average BER versus coefficient of powers.

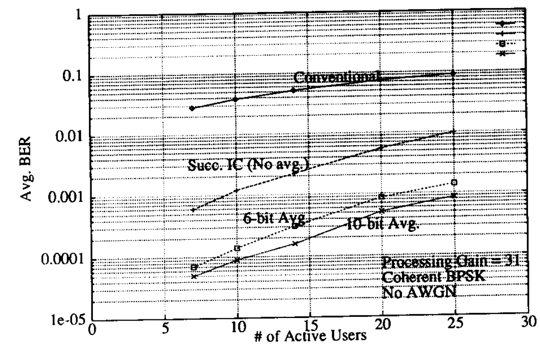

Fig. 9. Average BER versus number of active users, with no averaging, 6–bit averaging, and 10–bit averaging.

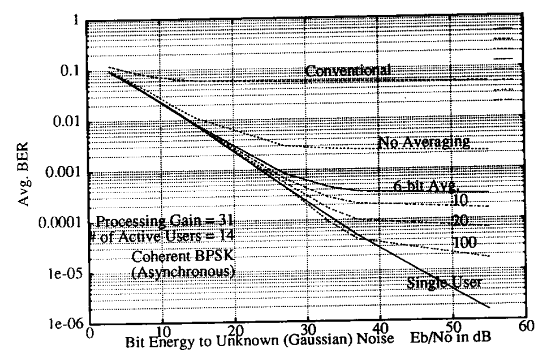

Fig. 10. Average BER versus E_b/N_o.

eral statement about its performance under fading with varying degrees. So to observe the effect of power unbalance on the BER performance of the IC scheme, we consider cases where the power levels of users vary with different coefficient of variations (CV). From Fig. 8, it is seen that with perfect power control ($CV = 0$), there is an improvement in going from conventional to IC but not as big an improvement as when CV is bigger (around 0.4). As variations increase, the performance of the IC scheme improves initially, and then starts degrading, whereas the performance of the conventional receiver starts degrading as soon as power unbalance is introduced.

D. Effect of Averaging over n Bits on the Performance of IC Scheme

As described before, average correlations over n bits was used to rank the users but the estimated averaging did not use averaging. In this section, we shall study the performance improvement achieved by using average correlations to estimate the amplitude. As a matter of fact, by using the average correlations over n bits, the variance of the noise in the estimate of the amplitude decreases by a factor of $1/n$ (assuming that the correlations are independent from bit to bit). Using (6) and the above fact, we obtain the variance of the noise in the decision variable after jth cancellation as (14), see below. where n is the number of bits used for averaging. We shall now study the effect of averaging over n bits on the performance of IC Scheme under Rayleigh fading.

Fig. 9 is the plot of the average BER versus the number of active users. As the estimation of the amplitude of the bit is improved by averaging over several bits, the BER performance improves significantly in a fading environment. The improvement in the average BER (and hence the capacity) is evident from Fig. 9 as 6-bit or 10-bit averaging is used instead of no averaging. It is also clear from Fig. 10 that as the estimate of the amplitude of the user being cancelled is improved (by increasing the number of bits being averaged), the single user bound can be approached in a fading channel. This, however, assumes that the amplitude is fixed over the averaging interval which does not account for fast Rayleigh fading[6].

Note on Using Power Estimates Instead of Correlations An alternative to using correlation values to determine the order of cancellation and to perform cancellation is to use separate channel estimates. First-order analysis of the IC scheme using power estimates instead was done in [13], which was also verified by simulations. It was noted that for comparable BER performance the power estimate must be accurate within 3 dB of the actual power value, if correlation of just one bit is used. However, if correlations over 10 bits are averaged, the required accuracy of the power estimate becomes more strict (around 1 dB).

[6] For example, the Rayleigh fading amplitude correlation between a received symbol and the tenth succeeding symbol is around 0.94 for frequency of 1.8 GHz and symbol rate of 30 k symbols/s and 100 km/h velocity.

$$E_{A_k}[\eta_{j+1}] = \begin{cases} \frac{1}{N}\sum_{k=j+2}^{K} E[A_k^2] + \frac{N_o}{T} + \frac{1}{Nn}\sum_{i=1}^{j}\eta_i; & \text{for Synchronous} \\ \frac{1}{3N}\sum_{k=j+2}^{K} E[A_k^2] + \frac{N_o}{T} + \frac{1}{3Nn}\sum_{i=1}^{j}\eta_i; & \text{for Asynchronous} \end{cases} \tag{14}$$

Until now, we derived a method for analyzing a successive interference cancellation scheme in a DS/CDMA system using coherent BPSK modulation. The analysis was verified through simulations. Significant performance improvement over the conventional receiver was demonstrated, for both under ideal power control and under fading. This translates to increased capacity and fading resistance. It was also shown that averaging significantly improves the performance of the IC scheme and that the single-user bound can be achieved. In the next section, we shall extend our analysis to a more robust modulation scheme for mobile radio, namely M-ary orthogonal modulation. Here, the IC scheme is applied to the noncoherent receiver, and its performance with multipath diversity combining is studied.

IV. IC Scheme with M-ary Orthogonal Modulation

A. System Model

A CDMA system is under implementation as a next generation digital cellular system. In this system [22], a combination of orthogonal signalling and code division multiple access is used on the reverse link (mobile to base) to overcome the unavailability of a pilot signal providing coherent reference. This scheme is efficient in providing non coherent detection of CDMA signals. On the reverse link, 64 Walsh functions are used to obtain 64-ary orthogonal modulation. Fig. 11 shows the block diagram of the modulation scheme. The received version of the transmitted Offset-QPSK (OQPSK) signals from all the mobiles in a single cell during one symbol interval T_w is given by

$$r(t) = \sum_{k=1}^{K} S_{k,j}(t - \tau_k) + n(t) \tag{15}$$

Assuming that there are N multipaths per user that are arriving at the base station, the received signal is explicitly given by

$$\begin{aligned} r(t) = \sum_{k=1}^{K}\sum_{i=1}^{N}&[\alpha_k^i \sqrt{P_k} \cdot W_k^j(t - \tau_k^i) \cdot a_k((t - \tau_k^i) \\ &\times p_I(t - \tau_k^i)\cos(\omega_c t + \phi_k^i) \\ &+ \alpha_k^i \sqrt{P_k} \cdot W_k^j(t - T_d - \tau_k^i) \cdot a_k(t - T_d - \tau_k^i) \\ &\times p_Q(t - T_d - \tau_k^i)\sin(\omega_c t + \phi_k^i)] + n(t) \end{aligned} \tag{16}$$

$r(t)$ = received signal
K = total number of active users
P_k = power of each chip of k^{th} user
α_k^i = varying amplitude of the i^{th} multipath of the k^{th} user
$W_k^j(t)$ = j^{th} M-ary symbol of k^{th} user ($j = 1, \ldots, M$) (Walsh symbol)
$a_k(t)$ = spreading chip sequence of k^{th} user
$pI(t)$ = I channel short PN sequence
$p_Q(t)$ = Q channel short PN sequence
$n(t)$ = additive white Gaussian noise (two-sided power spectral density $= N_0/2$)

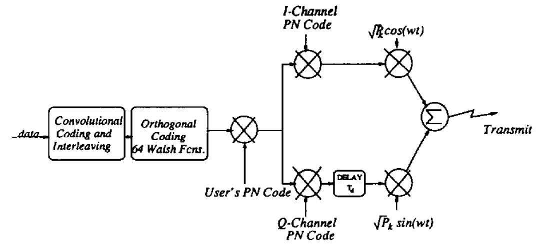

Fig. 11. DS/CDMA transmitter.

N = total number of multipaths
τ_k^i and ϕ_k^i are the time delay and phase of the i^{th} multipath of the k^{th} user.
τ_k are assumed to be known, i.e., tracked accurately.
T_c = chip interval
T_d = offset time ($T_c/2$)

The τ_k^i and ϕ_k^i are i.i.d. uniform random variables in $[0, T]$ and $[0,\ 2\pi]$, respectively, for the asynchronous case. No knowledge of ϕ_k^i is assumed.

For simplicity of notation, we shall denote the product of the M-ary symbol, the user PN code, and the I or Q channel PN code as follows

$$\begin{aligned} a_{k,j}^I(t) &= W_k^j(t) \cdot a_k(t) \cdot p_I(t) \\ a_{k,j}^Q(t) &= W_k^j(t - T_d) \cdot a_k(t - T_d) \cdot p_Q(t - T_d) \end{aligned} \tag{17}$$

Rewriting the received signal, we get

$$\begin{aligned} r(t) = \sum_{k=1}^{K}\sum_{i=1}^{N} \quad &[\alpha_k^i \sqrt{P_k} \cdot a_{k,j}^I(t - \tau_k^i) \cdot \cos(\omega_c t + \phi_k^i) \\ &+ \alpha_k^i \sqrt{P_k} \cdot a_{k,j}^Q(t - \tau_k^i).\sin(\omega_c t + \phi_k^i)] + n(t) \end{aligned} \tag{18}$$

We assume knowledge of the spread sequences of all the users, but no knowledge of the powers and phases of the individual users is needed. As shown in Fig. 12, the receiver correlates the received signal with the respective I & Q channel PN code and with the respective user's spreading code and with all possible 64-ary symbols for all the tractable multipaths. Each multipath that is tracked generates 64 coefficients representing the 64 possible Walsh symbols. These 64 coefficients from all the multipaths are then combined to produce a single set of 64 coefficients. A decision on the symbol is then made by selecting the largest of these 64 coefficients. This is the conventional CDMA receiver for noncoherent reception. Analysis of this receiver in the AWGN channel was done in [23]. Analysis of M-ary modulation in multipath channel was done in [24] for Rayleigh distributed multipaths and for general multipaths in [25]. We shall follow the analysis of [25] with multipath combining and apply Interference Cancellation to it. Note again that we are not considering convolutional coding and interleaving here. At the output of the lowpass filter (LPF) of the I channel, we get

$$\begin{aligned} d^I(t) &= \mathrm{LPF}\{r(t)\cos(\omega_c t)\} \\ &= \sum_{k=1}^{K}\sum_{i=1}^{N}\left\{\left[\alpha_k^i\sqrt{P_k}\cdot a_{k,j}^I(t-\tau_k^i)\cdot\frac{\cos(\phi_k^i)}{2}\right]\right. \\ &\quad \left.+\left[\alpha_k^i\sqrt{P_k}\cdot a_{k,j}^Q(t-\tau_k^i)\cdot\frac{\sin(\phi_k^i)}{2}\right]\right\}+\frac{n_c(t)}{2} \end{aligned} \tag{19}$$

where $n_c(t)$ is the in-phase component of the Gaussian noise ($n(t)$ after lowpass filtering can be represented by $n_c(t) + j\cdot n_s(t)$). Similarly, $d^Q(t)$ is obtained. As seen in Fig. 12, the decision variable obtained from the l^{th} multipath of the k^{th} is given by $S_{k,m}(l)$; $m = 1,\dots M$. Combining all L multipaths, we obtain $S_{k,m} = \sum_{l=1}^{L} S_{k,m}(l)$ and the decision on the symbol of the k^{th} user is then obtained as $\hat{m} = \max_m [S_{k,m}]; m = 1,\dots M$. For a particular user (say user 1), we have the following

$$S_{1,m}(l) = \left(Z_{1,m}^{II}(l) + Z_{1,m}^{QQ}(l)\right)^2 + \left(Z_{1,m}^{IQ}(l) - Z_{1,m}^{QI}(l)\right)^2 \tag{20}$$

where $Z_{1,m}^{II}(l), Z_{1,m}^{IQ}(l), Z_{1,m}^{QI}(l)$ and $Z_{1,m}^{QQ}(l)$ are the respective correlators outputs for the m^{th} symbol of the 1^{st} user from the l^{th} multipath and are given by (22)–(24), see below. Similarly, we get Here, $I_1^{II}, I_1^{IQ}(l), I_1^{QQ}(l)$, and $I_1^{QI}(l)$ are the respective interferences due to all multipaths of other (K–1) users. $I_{1,1}^{II}(l), I_{1,1}^{IQ}(l), I_{1,1}^{QQ}(l)$, and $I_{1,1}^{QI}(l)$ are the self-interferences due to a user's own multipaths. $N_1^{II}(l), N_1^{IQ}(l), N_1^{QQ}(l)$, and $N_1^{QI}(l)$ are the respective thermal noises which are considered to be uncorrelated Gaussian random variables. In the next subsection, we shall apply the IC scheme.

B. Interference Cancellation

The IC scheme is a simple successive scheme (as described before), where the strongest user (hence the most reliable one) is decoded first and cancelled from the composite signal. The process is repeated until all the users are decoded. Fig. 13 is the schematic of the DS/CDMA receiver with interference cancellation and Rake receiver for multipath diversity combining, where for the strongest user each multipath that is being tracked by the rake is cancelled from the received signal (at baseband) by using the appropriate correlator outputs. The maximum decision variable output is selected as the strongest user. Say user 1 was the strongest user (selected as $\max_k S_{k,j}$). After decoding the j^{th} symbol (picked as $\max_m (S_{1,m}); m = 1,\dots M$ of user 1, its signal (all the multipaths of user 1) is regenerated and cancelled from the respective $I\&Q$ components at the baseband level. After making the decision on its symbol, all the L multipaths of user 1 are cancelled from the received signal (at baseband) as follows

$$\begin{aligned} d_1^I(t) = d^I(t) &- \sum_{l=1}^{L} Z_{1,j}^{II}(l)\cdot a_{1,j}^I(t-\tau_1^l) \\ &- \sum_{l=1}^{L} Z_{1,j}^{IQ}(l)\cdot a_{1,j}^Q(t-\tau_1^l) \end{aligned} \tag{25}$$

Note that, out of the N multipaths that are reaching the base station, only L multipaths which are being tracked by the Rake receivers are being cancelled.

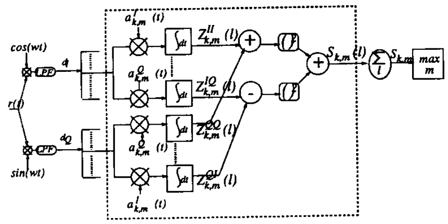

Fig. 12. Noncoherent DS/CDMA receiver.

$$\begin{aligned} Z_{1,m}^{II}(l) &= \frac{1}{T_w}\int_0^{T_w} d^I(t)a_{1,m}^I(t-\tau_1^l)dt \\ &= \begin{cases} \alpha_1^l\sqrt{P_1}\frac{\cos(\phi_1^l)}{2} + I_{1,1}^{II}(l) + I_1^{II}(l) + N_1^{II}(l)\ ; \text{ if } m = j \\ I_{1,1}^{II}(l) + I_1^{II}(l) + N_1^{II}(l)\ ; \text{ else} \end{cases} \end{aligned} \tag{21}$$

$$Z_{1,m}^{IQ}(l) = \begin{cases} \alpha_1^l\sqrt{P_1}\frac{\sin(\phi_1^l)}{2} + I_{1,1}^{IQ}(l) + I_1^{IQ}(l) + N_1^{IQ}(l)\ ; \text{ if } m = j \\ I_{1,1}^{IQ}(l) + I_1^{IQ}(l) + N_1^{IQ}(l)\ ; \text{ else} \end{cases} \tag{22}$$

$$Z_{1,m}^{QQ}(l) = \begin{cases} \alpha_1^l\sqrt{P_1}\frac{\cos(\phi_1^l)}{2} + I_{1,1}^{QQ}(l) + I_1^{QQ}(l) + N_1^{QQ}(l)\ ; \text{ if } m = j \\ I_{1,1}^{QQ}(l) + I_1^{QQ}(l) + N_1^{QQ}(l)\ ; \text{ else} \end{cases} \tag{23}$$

$$Z_{1,m}^{QI}(l) = \begin{cases} -\alpha_1^l\sqrt{P_1}\frac{\sin(\phi_1^l)}{2} + I_{1,1}^{QI}(l) + I_1^{QI}(l) + N_1^{QI}(l)\ ; \text{ if } m = j \\ I_{1,1}^{QI}(l) + I_1^{QI}(l) + N_1^{QI}(l)\ ; \text{ else} \end{cases} \tag{24}$$

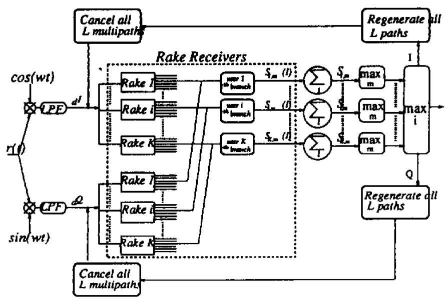

Fig. 13. Noncoherent DS/CDMA receiver with interference cancellation and multipath combining.

Simplifying this expression, we get

$$d_1^I(t) = \sum_{k=2}^{K}\sum_{l=1}^{N} s_{k,j,l}(t-\tau_k) + \frac{n_c(t)}{2} + \text{noise due to cancellation} \tag{26}$$

where $s_{k,j,l}(t-\tau_k)$ is the signal from l^{th} multipath of the k^{th} user which is not yet decoded and cancelled.

Explicitly,

$$\begin{aligned} d_1^I(t) = &\sum_{k=2}^{K}\sum_{l=1}^{N} s_{k,j,l}(t-\tau_k^l) + \frac{n_c(t)}{2} \\ &+ \sum_{l=L+1}^{N} s_{1,j,l}(t-\tau_1^l) \\ &- C_1^I a_{1,j}^I(t-\tau_1^l) - C_1^Q a_{1,j}^Q(t-\tau_1^l) \end{aligned} \tag{27}$$

In this expression, the first term is the signals from all the multipaths of all other remaining $K-1$ users, the second term is the in-phase component of Gaussian noise, the third term is the contribution of the uncancelled multipaths of user # 1, and the last two terms are the noise due to imperfect cancellation, where C_1^I and C_1^Q are as follows:

$$C_1^I = \sum_{l=1}^{L}\{I_{1,1}^{II}(l) + I_1^{II}(l) + N_1^{II}(l)\}$$

and

$$C_1^Q = \sum_{l=1}^{L}\{I_{1,1}^{IQ}(l) + I_1^{IQ}(l) + N_1^{IQ}(l)\} \tag{28}$$

Similarly, $d_1^Q(t)$ is obtained.

Now for the next strongest user, there are only $K-2$ interferers (since we effectively cancelled out the strongest user, though not perfectly). The correlations for the next strong user (user #2) are given by

$$\begin{aligned} Z_{2,m}^{II}(l) &= \frac{1}{T_w}\int_{\tau_2^l}^{\tau_2^l+T_w} d_1^I(t) a_{2,m}^I(t-\tau_2^l)\,dt \\ &= \begin{cases} \alpha_2^l\sqrt{P_2}\frac{\cos(\phi_2^l)}{2} + C_2^I(l); & \text{if } m=j \\ C_2^I(l); & \text{else} \end{cases} \end{aligned} \tag{29}$$

where $C_2^I(l)$ is as follows

$$\begin{aligned} C_2^I(l) = &I_{2,2}^{II}(l) + I_2^{II}(l) + N_2^{II} - C_1^I I_{1,2} \\ &- C_1^Q I_{1,2} + R_1^I + R_1^Q \end{aligned} \tag{30}$$

where $R_1^{I/Q}$ is the noise due to the uncancelled multipaths of user 1 and are given by

$$R_1^{I/Q} = \sum_{l=L+1}^{N} \alpha_1^l\sqrt{P_1}\cdot I_{1,2}\cdot\frac{\cos(\phi_1^l)}{2} \tag{31}$$

$I_{1,2}$ is defined as follows

$$I_{1,2} = \frac{1}{T_w}\int_{\tau_1}^{\tau_1+T_w} a_{1,m}^{I/Q}(t-\tau_1)\cdot a_{2,j}^{I/Q}(t-\tau_2)\,dt \tag{32}$$

Similarly, $Z_{2,m}^{IQ}(l)$, $Z_{2,m}^{QI}(l)$, and $Z_{2,m}^{QQ}(l)$ are obtained. The decision variable for the second user is then obtained as

$$S_{2,m} = \sum_{l=1}^{L} S_{2,m}(l) \tag{33}$$

where

$$S_{2,m}(l) = \left(Z_{2,m}^{II}(l) + Z_{2,m}^{QQ}(l)\right)^2 + \left(Z_{2,m}^{IQ}(l) - Z_{2,m}^{QI}(l)\right)^2 \tag{34}$$

Now the second cancellation is given by

$$\begin{aligned} d_2^I(t) = d_1^I(t) &- \sum_{l=1}^{L} Z_{2,j}^{II}(l).a_{2,j}^I(t-\tau_2^l) \\ &- \sum_{l=1}^{L} Z_{2,j}^{IQ}(l)\cdot a_{2,j}^Q(t-\tau_2^l) \end{aligned} \tag{35}$$

Hence, following the same procedure, where for the h^{th} cancellation we have

$$\begin{aligned} d_h^I(t) = d_{h-1}^I(t) &- \sum_{l=1}^{L} Z_{h,j}^{II}(l)\cdot a_{h,j}^I(t-\tau_h^l) \\ &- \sum_{l=1}^{L} Z_{h,j}^{IQ}(l)\cdot a_{h,j}^Q(t-\tau_h^l) \end{aligned} \tag{36}$$

Simplifying, we get

$$\begin{aligned} d_h^I(t) = &\sum_{k=h+1}^{K}\sum_{l=1}^{N} s_{k,j,l}(t-\tau_k^l) + \frac{n_c(t)}{2} \\ &+ \sum_{k=1}^{h}\sum_{l=L+1}^{N} s_{k,j,l}(t-\tau_k^l) - \sum_{i=1}^{h}\{C_i^I a_{i,j}^I(t-\tau_i^l) \\ &+ C_i^Q a_{i,j}^Q(t-\tau_i^l) \end{aligned} \tag{37}$$

where $C_i^I = \sum_{l=1}^{L} C_i^I(l)$ and $C_i^Q = \sum_{l=1}^{L} C_i^Q(l)$. Similarly, $d_h^Q(t)$ is obtained.

Hence, after h cancellations only $K-h$ users are remaining. The correlations for the $h+1^{\text{st}}$ user are obtained as follows:

$$\begin{aligned} Z_{h+1,m}^{II}(l) &= \frac{1}{T_w}\int_{\tau_{h+1}^l}^{\tau_{h+1}^l+T_w} d_h^I(t) a_{h+1,m}^I(t-\tau_{h+1}^l)\,dt \\ &= \begin{cases} \alpha_{h+1}^l\sqrt{P_{h+1}}\frac{\cos(\phi_{h+1}^l)}{2} + C_{h+1}^I(l)\ ; & \text{if } m=j \\ C_{h+1}^I(l)\ ; & \text{else} \end{cases} \end{aligned} \tag{38}$$

where $C^I_{h+1}(l)$ is as follows

$$C^I_{h+1}(l) = I^{II}_{h+1,h+1}(l) + I^{II}_{h+1}(l) + N^{II}_{h+1} - \sum_{i=1}^{h}[C^I_i I_{1,2} + C^Q_i I_{1,2}] + \sum_{i=1}^{h}\left[R^I_i + R^Q_i\right] \tag{39}$$

In this expression, the first term is the self-interference of the $h+1^{\text{st}}$ user (which is negligible), the second term is the multiple access interference from the remaining K–h–1 users, the third term is the thermal (Gaussian) noise, and the last two terms are the noise due to imperfect cancellations in the previous stages.

Further analysis of the terms in (39) yields the following (a detailed analysis can be found in the Appendix). Denoting the variance of the noise in the correlator output as follows, we get

$$\begin{aligned}\eta^I_{h+1}(l) &= \mathrm{Var}\left[Z^{II}_{h+1\cdot m}(l)\right] \\ &= \frac{N_o}{4T_w} + \frac{1}{6N_c}\sum_{\substack{n=1 \\ n\langle\rangle l}}^{N} P_{h+1}\left(\alpha^l_{h+1}\right)^2 \\ &\quad + \frac{1}{6N_c}\sum_{k=h+2}^{K}\sum_{n=1}^{N} P_k(\alpha^n_k)^2 + \frac{4}{3N_c}\sum_{i=1}^{h} L\cdot\eta^I_i(l) \\ &\quad + \frac{1}{6N_c}\sum_{k=1}^{h}\sum_{n=L+1}^{N} P_k(\alpha^n_k)^2\end{aligned} \tag{40}$$

where N_c is the chips/symbol. Conditioned on α^n_k, $\left(Z^{II}_{h+1,m}(l) + Z^{QQ}_{h+1,m}(l)\right)$ is approximated as a Gaussian random variable with variance η_{h+1}, given by

$$\begin{aligned}\eta_{h+1} &= \eta^I_{h+1}(l) + \eta^Q_{h+1}(l) = 2\eta^I_{h+1}(l) \\ &= \frac{N_o}{2T_w} + \frac{1}{3N_c}\sum_{\substack{n=1 \\ n\neq l}}^{N} P_{h+1}\left(\alpha^n_{h+1}\right)^2 \\ &\quad + \frac{1}{3N_c}\sum_{k=h+2}^{K}\sum_{n=1}^{N} P_k(\alpha^n_k)^2 + \frac{4}{3N_c}\sum_{i=1}^{h} L\eta_i \\ &\quad + \frac{1}{3N_c}\sum_{k=1}^{h}\sum_{n=L+1}^{N} P_k(\alpha^n_k)^2\end{aligned} \tag{41}$$

Similarly, $\left(Z^{IQ}_{h+1,m}(l) - Z^{QI}_{h+1,m}(l)\right)$ is also a Gaussian random variable with variance η_{h+1}. Hence, $S_{h+1,m}(l)$ becomes a Chi-square distributed random variable with two degrees of freedom. Assuming that the α^k_n are i.i.d. random variables, then the variance $\eta_h + 1$ is independent of l (i.e., same for all multipaths). Therefore, the decision variable of the $h+1^{\text{st}}$ user becomes a Chi-square distributed random variable with $2L$ degrees of freedom. The recovered power of the $h+1^{\text{st}}$ user by tracking L multipaths is given by

$$c^2_{h+1} = \sum_{l=1}^{L} P_{h+1}\left(\alpha^l_{h+1}\right)^2 \tag{42}$$

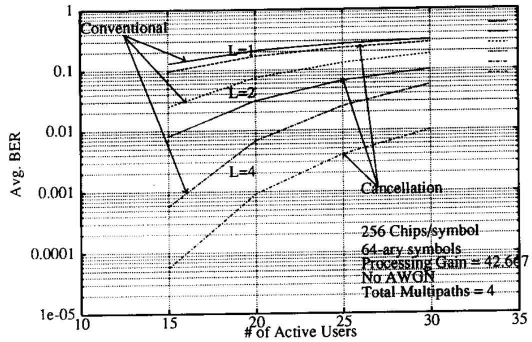

Fig. 14. Average bit error rate versus number of active users under ideal power control.

The probability of correct decision of the $h+1^{\text{st}}$ user is given by [26], [25]

$$P^{h+1}_c \int_0^\infty \left[1 - e^{-\frac{s_1}{2\eta_{h+1}}}\sum_{l=0}^{L-1}\frac{s^l_1}{(2\eta_{h+1})^l l!}\right]^{M-1} \cdot \frac{1}{(2\eta_{h+1})} \cdot \left(\frac{s_1}{c^2_{h+1}}\right)^{\left(\frac{L-1}{2}\right)} \cdot e^{-\frac{c^2_{h+1}+s_1}{2\eta_{h+1}}} \cdot I_{L-1}\left(\frac{\sqrt{c^2_{h+1}s_1}}{\eta_{h+1}}\right).ds_1 \tag{43}$$

where $I_L(.)$ is the modified Bessel function of the L^{th} order. The bit error probability, *conditioned on* c^2_{h+1} is then given by

$$P^{h+1}_b = \frac{2^{d-1}}{M-1}(1 - P^{h+1}_c) \tag{44}$$

where $d = \log_2 M$.

C. Performance with Multipath Combining

Fig. 14 is obtained by numerically evaluating (43), (44) for ideal power control (i.e., c^2 are equal for all users). As seen in the figure, the performance of both the conventional receiver and the IC scheme improves as the number of multipaths tracked (L) increases. Also note that the performance improvement of the IC scheme over the conventional receiver also increases as the number of multipaths tracked increases.

Note that the expression for bit error, after the h cancellations, is conditioned on the ordered set of c^2. We shall use order statistics (as described before) to obtain the average bit error rate under fading. As c^2's are the combined mean squares of the amplitudes of all multipaths that are being tracked, its distribution depends on the distribution of the α_k's. Assuming α_k's to be lognormal, we can make the approximation that c^2 (which is a sum of lognormal r.v's) is also lognormal (this was justified in [27]). We, thus, consider c^2 to be lognormally distributed with mean μ and variance σ^2. The pdf and cdf of c^2 is, hence, $f(x) = \frac{1}{\sqrt{2\pi}\xi x}e^{-\frac{(\log(x)-\zeta)^2}{2\sigma^2}}$ and $F(x) = \Phi\left(\frac{\log(x)-\zeta}{\xi}\right)$, respectively. ζ and ξ are the mean and standard deviation of the normal distribution from which the lognormal distribution is obtained. The pdf of ordered c^2_k, $f_{c^2_k}(x)$, is then obtained as described earlier [using (11)].

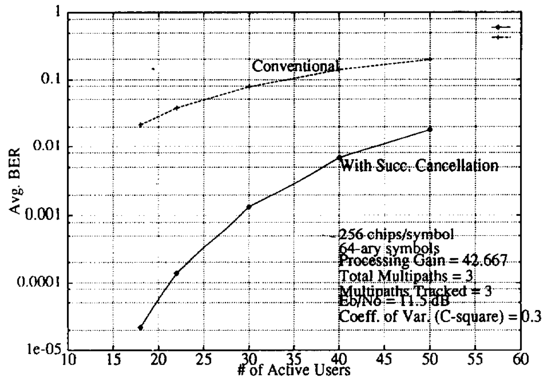

Fig. 15. Average bit error rate versus number of active users under fading.

Denoting the mean and standard deviation of the k^{th} strongest c^2 as μ_k and σ_k, we can obtain the average BER at the k^{th} cancellation stage, using the following approximation

$$\widehat{P_b^k} \approx \frac{2}{3} P_b^k(\mu_k) + \frac{1}{6} P_b^k(\mu_k + \sqrt{3}\sigma_k) + \frac{1}{6} P_b^k(\mu_k - \sqrt{3}\sigma_k) \tag{45}$$

This approximation has shown good accuracy in other applications (see [28]) and, more importantly, for a closely related problem with lognormal random variables [25]. For simplifying the analysis, we shall assume that there are three resolvable multipaths and all three paths are being tracked. The combined energy c^2 is assumed to be lognormally distributed with the coefficient of variation $\left(cv = \left(\frac{\sigma}{\mu}\right)\right)$ equal to 0.3. In Fig. 15, we have plotted the average BER versus the number of active users. The performance improvement of the IC scheme over the conventional scheme is evident.

D. Issues in Practical Implementations

There are two different ways of implementing the successive IC scheme, as seen in Fig. 16. One way is to successively regenerate the strong user's signal using the output of the correlator and its chip sequence, and to cancel it from the received signal and proceed likewise. An alternative way to perform this cancellation is to obtain the cross correlation (by using separate integrators which are correlating only the chip sequences of the respective users at their respective timings) and then update the output of the correlator by using the cross correlation information. Thus, this method operates at the bit level. Both these methods are equivalent analytically. The number of operations required by both methods are the same. The difference is that, in the former method, the received signal itself is being updated (as signals are being cancelled) while in the latter method the output of the correlators are being updated. The latter method has the flexibility of performing the added integrations (cross correlations) separately, whereas the former method performs the additional integrations as the users' are decoded and cancelled successively. The advantages and disadvantages of each method are being studied.

We shall assume for now that the received signal is being successively updated as the strongest user is being cancelled. We shall consider that each Rake receiver has correlators

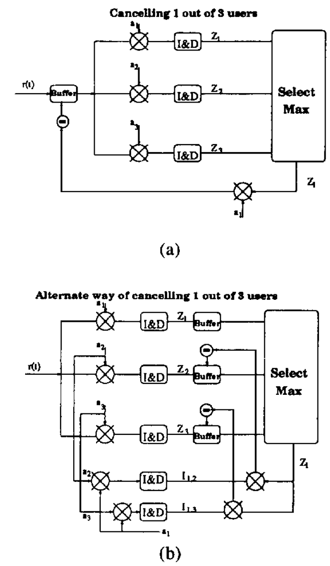

Fig. 16. Two methods of implementing successive IC schemes.

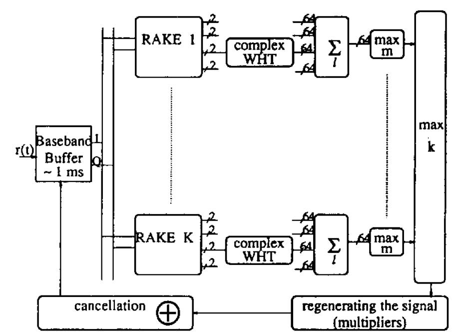

Fig. 17. Interference cancellation and multipath combining.

in parallel for tracking each multipath. A complex Walsh-Hadamard Transform (WHT) generates 64 coefficients for each multipath, corresponding to the 64 possible symbols. As can be seen from Fig. 17, for each user, each multipath that is being tracked has a Walsh decoder. The output from the WHT's from each multipath are then combined coherently, resulting in a single set of 64 coefficients. The maximum of these 64 coefficients determines the symbol sent by that particular user. Until this point, the receiver is the conventional DS/CDMA receiver.

The signal of the strongest user is then decoded and regenerated for cancellation. Note that the process of regenerating the signal and cancellation requires simple multipliers and adders. The processing delay is mainly limited by the speed of performing one Walsh-Hadamard Transform (WHT). Since successive cancellation is involved, the possible number of cancellations is limited by the speed of performing one WHT. In order to ensure a regular flow of symbols at the symbol rate R_s, the speed of the WHT must be at least $K \cdot R_s$ (where K is the possible number of cancellations). For example, in order to have at least 100 cancellations, the speed of the WHT must be at least 0.16 MHz (assuming a bit rate of 9.6

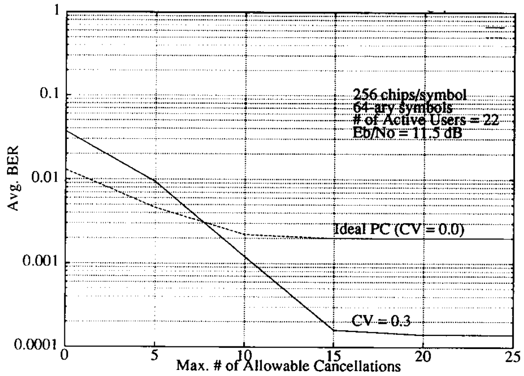

Fig. 18. Average bit error rate versus number of active users under fading.

kb/s), i.e., each WHT should take less than 6.25 μs. Thus, the processing speed of the hardware may limit the number of possible cancellations. The impact of limiting the number of cancellation on the BER performance of the IC scheme is depicted in Fig. 18. As the number of cancellations increases, the average BER decreases. Hence, there is a tradeoff of complexity versus performance. Faster processing implies a higher number of allowable cancellations which implies better BER performance.

V. Conclusion

Through this work, we have shown that by using a simple successive IC scheme, one can effectively estimate and cancel a CDMA signal and thus substantially reduce near/far effects from a CDMA system and increase the system capacity. The performance of the IC scheme with multipath diversity combining was also studied. Further work needs to be done in analyzing the processing delay involved and the practical implementation of the interference canceller as well as sensitivities to errors (such as delay tracking and quantization).

Appendix

A. Noise Analysis

In (39), the thermal noise $N_{h+1}^{II}(l)$ is expressed as follows

$$\begin{aligned} N_{h+1}^{II}(l) &= \frac{1}{2T_w}\int_{\tau_{h+1}^2}^{T_w+\tau_{h+1}^2} n_c(t)a_{h+1,j}^I(t-\tau_{h+1}^l)\,dt \\ &= \frac{1}{2T_w}\sum_{u=0}^{N_c-1}\int_{uT_c+\tau_{h+1}^2}^{(u+1)T_c+\tau_{h+1}^2} n_c(t)a_{h+1,j}^I(t-\tau_{h+1}^l)\,dt \\ &= \frac{1}{2T_w}\sum_{u=0}^{N_c-1}\int_{uT_c+\tau_{h+1}^2}^{(u+1)T_c+\tau_{h+1}^2} \pm n_c(t)\,dt \end{aligned} \tag{46}$$

where $N_c = \frac{T_w}{T_c}$. It can be shown that $N_{h+1}^{II}(l)$ is a Gaussian random variable with zero mean and variance $\frac{N_o}{4T_w}$. Similarly, $N_{h+1}^{IQ}(l)$, $N_{h+1}^{QQ}(l)$, and $N_{h+1}^{QI}(l)$ are also uncorrelated Gaussian random variables with zero mean and variance $\frac{N_o}{4T_w}$.

B. Interference Analysis

In (39), the first term is the self-interference due to a user's own multipaths. The second term in (39) is the multiple access interference after h cancellations and is expressed as

$$\begin{aligned} I_{h+1}^{II}(l) &= \sum_{k=h+1}^{K}\sum_{n=1}^{N}\frac{\sqrt{P_k}\alpha_k^n}{2T_w} \\ &\times \int^{T_w+\tau_{h+1}^2}\Big\{\big[a_{h+1,m}^I(t-\tau_{h+1}^l)a_{k,j}^I(t-\tau_k^n)\cos(\phi_k^n)\big] \\ &+ \Big[a_{h+1,m}^I(t-\tau_{h+1}^l)a_{k,j}^Q(t-\tau_k^n)\sin(\phi_k^n)\Big]\Big\}dt. \end{aligned} \tag{47}$$

The integral is the asynchronous cross correlation of random binary sequences which can be characterized as Gaussian random variable with zero mean and variance $\frac{1}{3N_c}$. Therefore, conditioned on the α_k^n, this multiple access interference is approximated as Gaussian random variable with zero mean and variance

$$\begin{aligned} \mathrm{Var}\ [I_{h+1}^{II}(l)|\alpha_k^n] &= \sum_{k=h+2}^{K}\sum_{n=1}^{N}\frac{P_k(\alpha_k^n)^2}{4}\left[\frac{1}{3N_c}+\frac{1}{3N_c}\right] \\ &= \frac{1}{6N_c}\sum_{k=h+1}^{K}\sum_{n=1}^{N}P_k(\alpha_k^n)^2. \end{aligned} \tag{48}$$

The self-interference [first term in (39)] is given by (49), see below. Following similar steps, we get

$$\begin{aligned} \mathrm{Var}\ [I_{h+1,h+1}^{II}(l)|\alpha_{h+1}^n] &= \sum_{\substack{n=1\\n\neq l}}^{N}\frac{P_{h+1}(\alpha_{h+1}^n)^2}{4}\left[\frac{1}{3N_c}+\frac{1}{3N_c}\right] \\ &= \frac{1}{6N_c}\sum_{\substack{n=1\\n\neq l}}^{N}P_{h+1}(\alpha_{h+1}^n)^2. \end{aligned} \tag{50}$$

The term $I_{1,2}$, as defined in (32), is also a random variable with zero mean and variance

$$\begin{aligned} \mathrm{Var}[I_{1,2}] &= \mathrm{Var}\left[\frac{1}{T_w}\int_{\tau_1^2}^{T_w+\tau_1^2} a_{2,m}^I(t-\tau_2^l)a_{1,j}^I(t-\tau_1^l)\,dt\right] \\ &= \frac{2}{3N_c} \end{aligned} \tag{51}$$

The fourth term in (39) is

$$\sum_{i=1}^{h}(C_i^I+C_i^Q)I_{1,2} = \sum_{i=1}^{h}\sum_{l=1}^{L}(C_i^I(l)+C_i^Q(l))I_{1,2} \tag{52}$$

$$\begin{aligned} I_{h+1,h+1}^{II}(l) = \sum_{\substack{n=1\\n\neq l}}^{N}\frac{\sqrt{P_{h+1}}\alpha_{h+1}^n}{2T_w} &\times \int_{\tau_{h+1}^2}^{T_w+\tau_{h+1}^2}\Big\{\big[a_{h+1,m}^I(t-\tau_{h+1}^l)a_{h+1,j}^I(t-\tau_{h+1}^n)\cos(\phi_{h+1}^n)\big] \\ &+ \Big[a_{h+1,m}^I(t-\tau_{h+1}^l)a_{h+1,j}^Q(t-\tau_{h+1}^n)\sin(\phi_{h+1}^n)\Big]\Big\}dt. \end{aligned} \tag{49}$$

Denoting $\eta_i^I(l) = \mathrm{Var}[C_i^I(l)|\alpha_k^n]$, and since each multipath that is being tracked is independent of each other, $C_i^I(l)$ and $C_i^Q(l)$ are i.i.d random variables with zero mean and variance $\eta_i^I(l)$. Therefore,

$$\mathrm{Var}\left[\sum_{i=1}^{h}(C_i^I + C_i^Q)I_{1,2}\right] = \frac{4}{3N_c}\sum_{i=1}^{h} L\eta_i^I(l) \tag{53}$$

The last term in (39), which is the uncancelled multipath of the users already decoded, is given by

$$\sum_{i=1}^{h}(R_i^I + R_i^Q) = \sum_{i=1}^{h}\sum_{l=L+1}^{N}\left\{\sqrt{P_i}\alpha_i^l I_{1,2}\frac{\cos(\phi_i^l)}{2} + \sqrt{P_i}\alpha_i^l I_{1,2}\frac{\sin(\phi_i^l)}{2}\right\} \tag{54}$$

This term is also a random variable with zero mean and variance

$$\mathrm{Var}\left[\sum_{i=1}^{h}(R_i^I + R_i^Q)\right] = \frac{1}{6N_c}\sum_{i=1}^{h}\sum_{l=L+1}^{N} P_i(\alpha_i^l)^2 \tag{55}$$

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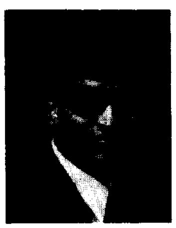

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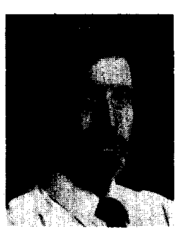

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