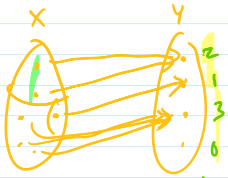
Addition Rule: for finite disjoint sets $X_1, \dots, X_m$

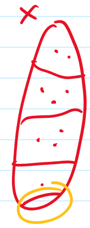
$$n\left(\bigcup_{i=1}^{m} X_i\right) = \sum_{i=1}^{m} n(X_i)$$



$n(X) = 2+1+3+0 = 6$

partition of X: a finite collection of sets X_i (part) s.t. $\bigcup X_i = X$

$X_i \cap X_j = \emptyset$ when $i \neq j$ and $X_i \neq \emptyset$



Ex: $A = \{1, 2, \dots, k\}$ $\quad B = \{(a,b) \in A \times A \mid a < b\}$

$= \{(1,2), (1,3), \dots, (1,k)$
$(2,3), (2,4), \dots (2,k)$
$(k-1, k)\}$

$n(B) = ??$

if $k=10$

$B_1 = \{(1,2), (1,3), (1,4), \dots, (1,10)\}$

$n(B_1) = 9$

$B_1 = \{(a,b) \in B \mid a = 1\}$

$B_2 = \{(a,b) \in B \mid a = 2\}$

$\vdots$

$B_k = \{(a,b) \in B \mid a = k\}$

$B = \bigcup_{i=1}^{k} B_i$

for general k, $n(B_1) = k-1$

$n(B_2) = k-2$

$\vdots$

$n(B_k) = 0$

$$n(B) = \sum_{i=1}^{k} n(B_i) = \sum_{i=1}^{k} k-i = \sum_{i=1}^{k} k - \sum_{i=1}^{k} i$$

$$k^2 - \frac{k(k+1)}{2}$$

$$= \frac{k(k-1)}{2}$$

$P(\text{roll a total of} \le 4 \text{ with 2 fair 6-sided dice})$

$n(E) = n(E_1) + n(E_2) + n(E_3)$

total = 2 $\quad$ total = 3 $\quad$ total = 4

$1 + 2 + 3 = 6$

$E = \{(x,y) \mid x+y=4 \wedge 1 \le x,y \le 6\}$

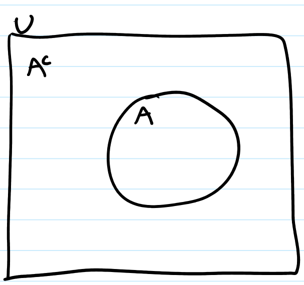
$$\frac{n(E)}{n(S)} \qquad \frac{6}{36} = \frac{1}{6}$$

outcomes w/ total 4

1) choose 1st die $\quad 3$

2) choose 4 − 1st die $\quad 1$

$3 \cdot 1$



$U = A \cup A^c \qquad A \cap A^c = \emptyset$

$n(U) = n(A) + n(A^c)$

$n(A^c) = n(U) - n(A)$

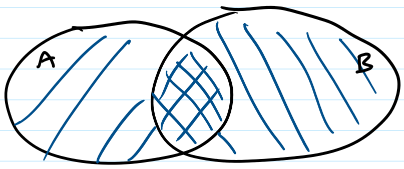
30 $\quad$ 5

Subtraction Rule

$P(\text{not rolling a total of } 7) = \frac{30}{36} = \frac{5}{6}$

$n(\text{total} \neq 7) = \text{total outcomes} - n(\text{total} = 7)$

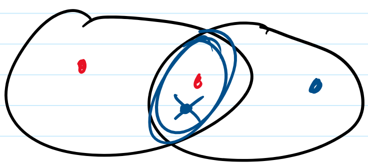
$= 36 - 6 = 30$



$A \cup B = (A-B) \cup (B-A) \cup (A \cap B)$

$A = (A-B) \cup (A \cap B)$

$B = (B-A) \cup (B \cap A)$



$n(A) = n(A-B) + n(A \cap B)$

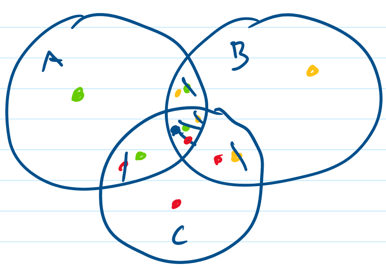
$n(B) = n(B-A) + n(A \cap B)$

$n(A) + n(B) = n(A-B) + n(B-A) + n(A \cap B) + n(A \cap B)$

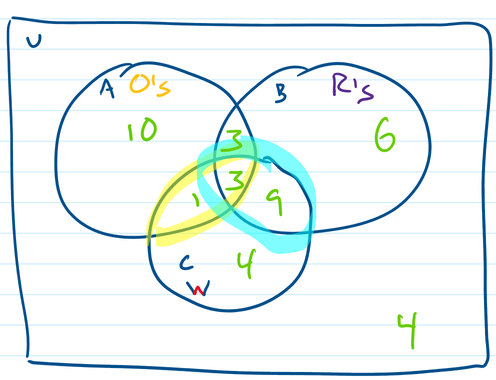
$n(A) + n(B) = n(A \cup B) + n(A \cap B)$

Inclusion / Exclusion:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



$n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(A \cap C)$
$- n(B \cap C)$
$+ n(A \cap B \cap C)$



total people	$n(U) = 40$
like Orioles	$n(A) = 17$
like Ravens	$n(B) = 21$
like both Orioles + Wizards	$n(A \cap C) = 4$
like O's or Ravens	$n(A \cup B) = 32$
like both R and W	$n(B \cap C) = 12$
like O's R's but not W	$n((A \cap B) - C) = 3$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$32 = 17 + 21 - n(A \cap B)$

$n(A \cap B) = 6$

like both Orioles & Wizards $n(A \cap C) = 4$

like O's or Ravens $n(A \cup B) = 32$

like both R and W $n(B \cap C) = 12$

like O's, Rs but not W $n((A \cap B) - C) = 3$

don't like any $n((A \cup B \cup C)') = 4$

$n(A \cap B) = 6$

We surveyed 40 people about what sports teams they follow. 17 follow at least the Orioles. 21 follow at least the Ravens. 4 follow at least both the Orioles and WIzards. 32 follow either the Ravens or Orioles. 12 follow at least both the Ravens and Wizards. 3 follow the Orioles and Ravens but not the Wizards. 4 follow none of the three teams. How many follow only the Wizards?

To solve the problem, we can use the principle of inclusion-exclusion (PIE) to count the number of people who follow only the Wizards.

Let's define the following sets:

- A = people who follow at least the Orioles
- B = people who follow at least the Ravens
- C = people who follow at least the Wizards

Using the given information, we can create a Venn diagram to represent the relationships between these sets:

cssCopy code

```
------------------- | C | ------------------- / \ / \ / \ / \ / \ / \ / \ / \ / A\ / B\ / \ / \ ------------------- ------------------- |
A∩ B∩ C | | (A∩ B) - C | ------------------- ------------------- | (A∩ C) - B| | (B∩ C) - A| ------------------- -------------------
| (B∩ C) - A| | (A∪ B∪ C)' | ------------------- ------------------- | (A∩ B) - C | | | ------------------- | (empty)
| -------------------
```

From the diagram, we can see that:

- A = (A ∩ B ∩ C) + (A ∩ B) + (A ∩ C) + (A ∩ !B ∩ !C)
- B = (A ∩ B ∩ C) + (A ∩ B) + (B ∩ C) + (!A ∩ B ∩ !C)
- C = (A ∩ B ∩ C) + (A ∩ C) + (B ∩ C) + (!A ∩ !B ∩ C)
- (A ∪ B ∪ C) = A + B + C - (A ∩ B) - (A ∩ C) - (B ∩ C) + (A ∩ B ∩ C)
- (A ∪ B ∪ C)' = number of people who do not follow any of the three teams

We are given the values of A, B, C, (A ∪ B ∪ C)', and we can find (A ∩ B ∩ C) as well:

- A = 17
- B = 21
- C = ?
- (A ∪ B ∪ C)' = 4
- (A ∩ B ∩ C) = 3

To find C, we can use the equation for B:

B = (A ∩ B ∩ C) + (A ∩ B) + (B ∩ C) + (!A ∩ B ∩ !C)

Substituting the values we know:

21 = 3 + (A ∩ B) + (B ∩ C) + (!A ∩ B ∩ !C)

We don't know (A ∩ B) or (!A ∩ B ∩ !C), but we can use the fact that (A ∪ B ∪ C) = 32 to eliminate them:

(A ∪ B ∪ C) = A + B + C - (A ∩ B) - (A ∩ C) - (B ∩ C) + (A ∩ B ∩ C)

32 = 17 + 21 + C - (A ∩ B) - (A ∩ !B ∩ !C) - (B ∩ C) + 3

Simplifying and rearranging:

(A

go on

Continuing from the previous message:

32 = 41 + C - (A ∩ B) - (B ∩ C)

(A ∩ B) + (B ∩ C) = 9

We can use this equation to eliminate (A ∩ B) from the equation for B:

21 = 3 + (A ∩ B) + (B ∩ C) + (!A ∩ B ∩ !C) 21 = 3 + 9 + (!A ∩ B ∩ !C) (!A ∩ B ∩ !C) = 9

Now we can use the equation for A to eliminate (A ∩ !B ∩ !C):

A = 17 = (A ∩ B ∩ C) + (A ∩ B) + (A ∩ C) + (A ∩ !B ∩ !C) 17 = 3 + (A ∩ B) + (A ∩ C) + 0 (A ∩ B) + (A ∩ C) = 14

Substituting these values back into the equation for (A ∪ B ∪ C):

32 = 17 + 21 + C - (A ∩ B) - (A ∩ C) - (B ∩ C) + (A ∩ B ∩ C) 32 = 38 + C - 9 - 14 - (B ∩ C) + 3 (B ∩ C) = 6

Finally, we can use the equation for C to find the number of people who follow only the Wizards:

C = (A ∩ B ∩ C) + (A ∩ C) + (B ∩ C) + (!A ∩ !B ∩ C) C = 3 + 14 + 6 + 4 = 27

Therefore, 27 people follow only the Wizards.

From <https://chat.openai.com/chat>

How many 5-card hands from 52-card deck?

order doesn't matter   3C 4H 9C JS KD
= KD 9C JS 4H 3C

r-combination of n things = size-r subset of n things

↓ how many 5-card subsets of 52 cards?

# of r-combos of n things = $\binom{n}{r}$ "n choose r" nCr $C(n,r)$ $\binom{52}{5}$ "52 choose 5" $52C5$ $C(52,5)$

Building an r-permutation: 3-perm of abcde: aed

a e d
1) choose 1st: n
2) choose 2nd: $n-1$
⋮
r) choose rth: $n-r+1$

$\binom{n}{r}$ 1) choose r in r-perm: {a, d, e}
$r!$ 2) permute them: 1st, 3rd, 2nd

r-permutations of n items $= n\cdot(n-1)\cdot\dots\cdot(n-r+1)$
$= \frac{n!}{(n-r)!} = \binom{n}{r}\cdot r!$

$$\binom{n}{r} = \frac{n!}{(n-r)!\cdot r!}$$

$$\binom{52}{5} = \frac{52!}{47!\cdot 5!}$$

$$\binom{n}{n-r} = \binom{n}{r}$$

$$\binom{n}{1} = n$$

$$\binom{n}{0} = 1$$

Poker 3 of a kind

$$\frac{n(\text{three kinds})}{n(\text{5-card hands})} = \frac{\quad}{\binom{52}{5}}$$

~~1) pick rank for 3K — 13~~
~~2) pick 3 of that rank — $\binom{4}{3} = \frac{4!}{3!\cdot 1!} = 4 = \binom{4}{1}$~~
~~3) pick 4th card — 48~~
~~4) pick 5th card — 44~~

3; H, C, S; 9D – KC; KC – 9D

3H 3C 3S 9D KC = 3H 3C 3S KC 9D   2 ways to get

3	1)	pick rank for 3K	13
{H, C, S}	2)	pick 3 of that rank	$\binom{4}{3}$
{9, K}	3)	pick 2 other ranks	$\binom{12}{2}$
D	4)	pick suit of lower other	4
C	5)	pick suit of last card	4

$$13\cdot\binom{4}{3}\cdot\binom{12}{2}\cdot 4\cdot 4$$

5H 3C 3J ... 5) pick suit of last card 4

$$13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4 \cdot 4$$

10 digit digital lock, 8 digit codes

Yahtzee small straight (but not large straight)

Pascals triangle

1

1 1

1 2 1

1 3 3 1

$\binom{4}{0} = 1 \quad \binom{4}{1} = 4 \quad \binom{4}{2} = 6$ 1 4 6 4 1

$\parallel$ 1 5 10 10 5 1 $\rightarrow \binom{5}{2}$

$\frac{4!}{2! \cdot 2!}$

1) include 1

2) include k-1 of other ~~n-1~~ $\{2, \dots, n\}$

$\{1, \dots, n\}$

\# size-k subsets $= \binom{n}{k}$

$=$ size-k subsets w/ 1 + size-k subsets w/o 1

$= \binom{n-1}{k-1} + \binom{n-1}{k}$

\# subsets

$$2^n = \sum_{k=0}^{n} \binom{n}{k}$$

Binomial coefficients

$$(x+y)^2 =$$

$(x+y)^5 =$

$(x+y)^6 = (x+y)(x+y)(x+y)(x+y)(x+y)(x+y)$

$(x+y)^n =$

Taste the Rainbow

| |000| |

strings of 0s 1s of length 7

3 0's 4 1's

| 0 | 0 | | 0

| 0 | 0 | 0 |

1) choose where 0's go

$\binom{7}{3} = \frac{7!}{3!4!} = \frac{7\cdot 6\cdot 5\cdot \cancel{4!}}{\cancel{4!}} = 35$

choosing r things
n ~~choices~~

$\binom{n-1+r}{r}$

	order matters	doesn't
repetition no		
yes