

Enumerative Combinatorics — the study of how to count things

Clique: Given n people and k , determine if there is a size- k subset who all know each other

$n=24$ 100
 $k=4$ 10

→ for each possible size- k subset
for all possible pairs in the subset
ask if the pair knows each other
if all answers were "YES", output the subset
output "NONE"

Chopsticks

P1		P2	
left	right	left	right
			2
2			2
	⋮		

how many different game states?

Probability

sample space S : set of all possible outcome

event E : subset of sample space

$S =$ result of drawing 1 card
 $= \{2C, 3C, \dots, AC, 2D, \dots, AD, 2S, \dots, AS, 2H, \dots, AH\}$

$E =$ draw black face card
 $= \{JC, QC, KC, JS, QS, KS\}$

$$E = \text{draw black face card} \\ = \{JC, QC, KC, JS, QS, KS\}$$

If all events in finite sample space S are equally likely, then

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(\text{draw black face card}) = \frac{6}{52} = \frac{3}{26}$$

$S =$ outcome of flipping 2 fair coins
 $E =$ exactly one head

$$S = \{0 \text{ heads}, 1 \text{ head}, 2 \text{ heads}\}$$

$$E = \{1 \text{ head}\}$$

~~$$P(\text{flipping 1 head}) = \frac{1}{3}$$~~

1980 penny

1947 penny

$$S = \{HH, HT, TH, TT\}$$

$$E = \{HT, TH\}$$

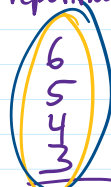
$$P(\text{flipping 1 head}) = \frac{2}{4} = \frac{1}{2}$$

$$E = \{11, 12, 13, 21, \dots, 33\}$$

Mechanical Lock: n buttons, must push all in correct order to unlock

how many possible ^{4 digits} combinations w/ 6 buttons?
 w/ (no repetition)

- 1) pick 1st digit
- 2) pick 2nd digit
- 3) pick 3rd
- 4) pick 4th



(every digit except one chosen in step 1)

$$6 \cdot 5 \cdot 4 \cdot 3 = 360$$

P(roll a straight with 6 6-sided dice)

E

$$S = \{111111, \dots, 666666\}$$

- 1) pick 1st thing
- 2) pick 2nd thing
- ...
- 6) pick 6th last

$$\rightarrow E = \{123456, \dots, 654321\}$$

permutation: ordering of things

$$n(S) = 6^6$$

6 factorial

$$n(E) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6^6} = 6!$$

$$P(\text{rolling straight}) = \frac{6!}{6^6}$$

Number of permutations of n things: $n!$

- 1) pick 1st
- ...
- n) pick last

$$\begin{matrix} n \\ \vdots \\ 1 \end{matrix} \quad \text{product} = n(n-1)(n-2)\dots \cdot 1 = n!$$

r -permutation: an ordering of a size- r subset

- 1) pick 1st
- ...
- r) pick r th

$$\begin{matrix} n \\ \vdots \\ n-r+1 \\ \vdots \\ 1 \end{matrix} \quad \left. \vphantom{\begin{matrix} n \\ \vdots \\ n-r+1 \\ \vdots \\ 1 \end{matrix}} \right\} \text{product} = \frac{n!}{(n-r)!}$$

10-button mechanical lock, 4-digit code

$$\# \text{ possible codes} = \# \text{ 4-permutations of 10 digits} = \frac{10!}{(10-4)!} = 10 \cdot 9 \cdot 8 \cdot 7$$

P(roll a total of ≤ 4 with 2 fair 6-sided dice)