Quantifier: a statement about size of truth set of a predicate

Universal $\forall$ true if truth set is entire domain

$\forall x \in \mathbb{Z},\ E(x) \vee O(x)$ T every integer is even or odd

($E(x)$: x is even; $O(x)$: x is odd; $\mathbb{Z}$: integers)

Existential $\exists$ true if truth set is non-empty

$\exists x \in \mathbb{Z},\ \underbrace{2 < x < 3}_{P(x)}$ F truth set is empty

T $\exists x \in \mathbb{R},\ 2 < x < 3$ truth set $= (2,3)$ $\{2.5, e, 2.841, \ldots\}$

($\mathbb{R}$: real number)

I \ R	Dunkin'	Starbucks	McDonald's	Five Guys	F(x)	B(x)
donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓

B(coffee)

$\exists x \in R$ s.t. $S(x, \text{burgers})$ T truth set $= \{McD, 5G\}$

$\exists x \in I$ s.t. $S(\text{Five Guys}, x)$ T truth set $= \{\text{burgers}, \text{cola}\}$

$\exists x \in I$ s.t. $\sim S(\text{McDonald's}, x)$ F truth set $= \{\ \}$

$\exists x \in I$ s.t. $F(x) \wedge S(\text{Starbucks}, x)$ T

Starbucks sells food.

$\forall x \in I,\ S(\text{McDonald's}, x)$ T

$\forall x \in R,\ S(x, \text{espresso})$ F truth set $= \{S, D, McD\}$

$\forall x \in I,\ F(x) \leftrightarrow \sim B(x)$

$\forall x \in R,\ S(x, \text{burgers}) \rightarrow S(x, \text{cola})$ T

Every restaurant that sells burgers also sells cola.

universal conditional

Every item sold at Starbucks is a beverage.

$\forall x \in I,\ S(\text{Starbucks}, x) \rightarrow B(x)$ F (counterex: x = donuts)

$S(x,y)$ = "x sells y"

$F(x)$ = "restaurant x is a food"

Starbucks only sells beverages.

Every restaurant that sells food sells espresso.

Every restaurant that sells food sells espresso.

$$\forall x \in R, \underbrace{\exists y \in I, S(x,y) \wedge F(y)}_{\text{something T of restaurants we're talking about}} \rightarrow \underbrace{S(x, \text{espresso})}_{\text{what we're saying about them}}$$

F: counterexample Five Guys

$\exists y \in I$ s.t. $\underbrace{S(FG,y) \wedge F(y)}$ is T

b/c $y =$ burgers makes this T

$S(FG, \text{espresso})$ F

Let $Q1(x) =$ " XYZ filed report x in Q1 "

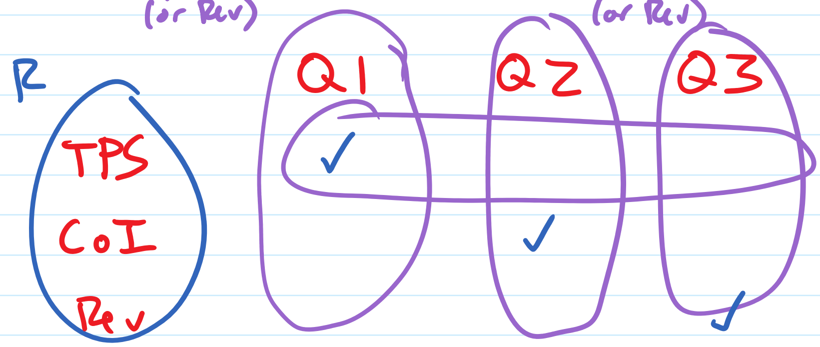
$Q2(x) =$ " XYZ filed report x in Q2 "

$Q3(x) =$ " XYZ filed report x in Q3 "

XYZ missed a report in all 3 quarters

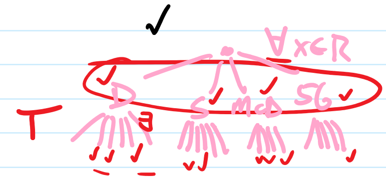
$$\rightarrow \exists x \in R \text{ s.t. } \left(\overset{F}{\sim Q1(x)} \wedge \overset{T}{\sim Q2(x)} \wedge \overset{T}{\sim Q3(x)} \right) \quad F$$

$$\underbrace{\exists x \in R \text{ s.t. } \sim Q1(x)}_{T\text{: example CoI (or Rev)}} \wedge \underbrace{\exists y \in R \text{ s.t. } \sim Q2(y)}_{T\text{: example TPS (or Rev)}} \wedge \underbrace{\exists z \in R \text{ s.t. } \sim Q3(z)}_{T\text{: example TPS (or CoI)}} \quad T$$



✓ means they filed report

	Dunkin'	Starbucks	McDonald's	Five Guys	F(x)	B(x)
donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓
D(x)	✓	✓	✓			



All restaurants sell beverages

$\forall x \in R,\ \exists y \in I$ s.t. $B(y) \wedge S(x,y)$

there is a beverage that x sells

work outside in: $\forall x \in R$, $P(x)$ is T exactly when every $x \in R$ makes $P(x)$ T

so check them all

$P(x)$ here is "$\exists y \in I$ s.t. $B(y) \wedge S(x,y)$"

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Dunkin}, y)$ T?

that is, is there a $y \in I$ that makes $B(y) \wedge S(\text{Dunkin'}, y)$ T?

the truth set for

yes, any of {coffee, espresso, cola}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Starbucks}, y)$ T? y truth set {coffee, espresso}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{McDonald's}, y)$ T? y truth set {coffee, espresso, cola}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Five Guys}, y)$ T? y

truth set {cola}

all $x \in R$ make $P(x)$ T, so $\forall x \in R, P(x)$ is T

$\forall x \in I,\ \exists y \in I$ s.t. $(F(x) \wedge \sim B(y)) \vee (\sim F(x) \wedge B(y))$

There's an item that all the drive-ins sell.

All the drive-ins have an item they sell in common.

$\forall x \in R,\ D(x) \rightarrow \exists y \in I$ ______

donuts

$\exists y \in I$ s.t. $\forall x \in R,\ D(x) \rightarrow S(x,y)$

all the drive-ins sell y

let y = donuts then $\forall x \in R,\ D(x) \rightarrow S(x, \text{donuts})$, is T

x = Starbucks	T → T	✓
x = Dunkin	T → T	✓
x = McD	T → T	✓
x = Five Guys	F → F	✓

Everyone sells everything.

$\forall x \in R, \forall y \in I, S(x,y) \equiv \forall y \in I, \forall x \in R, S(x,y)$

order doesn't matter when quantifier is of the same type

Everything McDonald's doesn't sell is a food.

Multiple Quantifiers

	Dunkin'	Starbucks	McDonald's	Five Guys	F(x)	B(x)
donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓

Something is sold at every store

Every store has some item they sell

$\forall x \in R, \underline{\exists y \in I \text{ s.t. } S(x,y)}$ T

There is an item common to all stores.

$\exists y \in I \text{ s.t. } \underline{\forall x \in R \quad S(x,y)}$ F

For a finite domain $D = \{\underline{x_1, x_2, \ldots, x_n}\}$

$$\forall x \in D,\ P(x) \equiv P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n)$$

$$\sim(\forall x \in D, P(x)) \equiv \sim(P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n))$$

$$\equiv \underline{\sim P(x_1)} \vee \underline{\sim P(x_2)} \vee \cdots \vee \underline{\sim P(x_n)}$$

$$\equiv \exists x \in D \text{ s.t. } \sim P(x)$$

$$\exists x \in D \text{ s.t. } P(x) \equiv \underline{P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)}$$

$$\sim \exists x \in D \text{ s.t. } P(x) \equiv \sim(\underline{P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)})$$

$$\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \cdots \wedge \sim P(x_n)$$

$$\equiv \underline{\forall} x \in D,\ \underline{\sim P(x)}$$

Some restaurant sells burgers

McDonald's sells everything

Everything McDonald's doesn't sell is a food.

$\forall x \in I,\ \sim S(\text{McDonald's}, x) \rightarrow F(x)$

All drive-ins have an item they sell in common

$\exists x \in I$ s.t. $\forall y \in R,\ S(y, x)$

All restaurants sell beverages.

All restaurants sell everything.

$\forall x \in R, \forall y \in I, S(x, y)$