Disingenuity

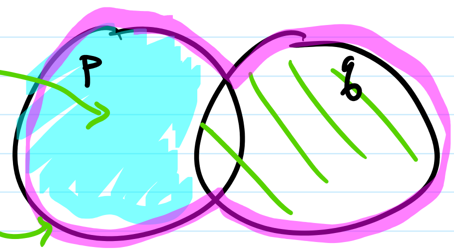
Alex has a rusty bike chain. p

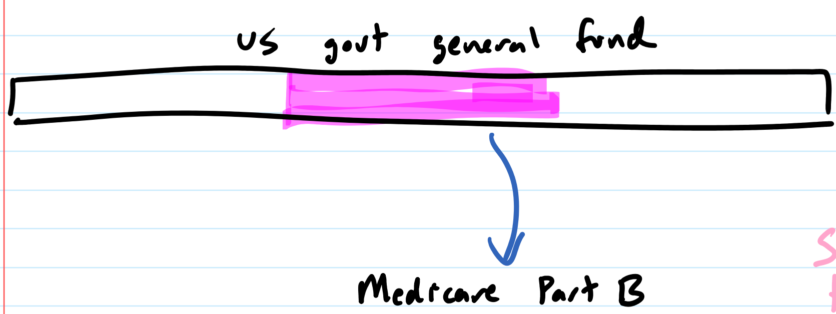
Alex or Haroon has a rusty bike chain. $\therefore p \lor q$

I know we are in

but I only tell you we are in

I didn't lie!





Medicare B take much $ from gen.

SS or Medicare B takes much $

Epp §2.3 Ex. 37

l = "The House is next to a lake"
k = "The treasure is in the kitchen"
e = "The tree in the front yard is an elm"
f = "The treasure is under the flagpole"
o = "The tree in the front yard is an oak"
g = "The treasure is in the garage"

$l \to \sim k$
$e \to k$
l
$e \vee f$
$o \to g$

Reymond Smullyan (via Epp)

A says "B is a knight"
B says "A and B are opposite"

$a \leftrightarrow b$

$b \leftrightarrow (a \leftrightarrow \sim b)$

1) Suppose A is a knight
2) A is telling truth
3) B is a knight — def knight what A said
4) B telling truth
5) A and B are opposite
6) B is not a knight
7) B is a knight $\wedge$ B not a knight
8) C
9) If A is a knight $\to$ C
10) ∴ A is not a knight
11) ∴ what A says is false
12) ∴ B is not a knight

A: Exactly 2 of us are knights
B: A is a knight
C: B is a knave or D is a knave
D: At least 3 of us are knights

Suppose B is a knave
Then A is not a knight so a knave
Then C is a knight
And D is a knave
If B is a knave, then C is the only knight

Suppose B is a knight
Then A is a knight
Then C, D are knaves
C is telling truth
C is a knight
C is a knave and C is a knight
C
If B is a knight $\to$ C
B is a knave

C is the only knight

A: I am a knight
B: I am a knight

Suppose P
∴ q

Suppose P
C

$\therefore q$

$p \to q$

c

$p \to c$

$\therefore \sim p$

Predicates

T 7 is odd
T 8 is even
F 9 is even

E(8)

statements

E(9)

x is even not a statement

predicate: statement w/ nouns (usually) removed, replaced w/ var

$E(x) =$ "x is even" $E(7) =$ "7 is even" F

$D(x) =$ " Dunkin' sells x "

domain

{ donuts, hash browns, coffee, espresso, burgers, cola }

$E(x) =$ " x sells espresso "

domain

{ Dunkin', Starbucks, McDonalds, Five Guys }

domain

truth set of F = {donuts, hash brow burgers}

$S(x,y) =$ " x sells y "

Dunkin' sells donuts

"x is food" "x is a beverage"

	Dunkin'	Starbucks	McDonald's	Five Guys	F(x)	B(x)
donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓

truth set of B = {coffee, espresso, cola}

truth set: set of elts in domain that make predicate true

for S = { (Dunkin', donuts), (Dunkin, hash browns), ... (Five Guys, cola) }

Quantifier: a statement about size of truth set of a predicate

Universal $\forall$ true if truth set is entire domain

$\forall x \in \mathbb{Z}, \; E(x) \vee O(x)$ T every integer is even or odd

(x is even; x is odd; $\mathbb{Z}$ ↑ integers)

Existential $\exists$ true if truth set is non-empty

$\exists x \in \mathbb{Z}, \; \underbrace{2 < x < 3}_{P(x)}$ F truth set is empty

T $\exists x \in \mathbb{R}, \; 2 < x < 3$ truth set $= (2, 3)$ $\{2.5, e, 2.841, \dots\}$

($\mathbb{R}$ ↑ real number)

	R: Dunkin'	Starbucks	McDonald's	Five Guys	F(x)	B(x)
I: donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓

$\exists x \in R$ s.t. $S(x, \text{burgers})$ T truth set $= \{McD, 5G\}$

$\forall x \in I, \; S(\text{McDonald's}, x)$ T

$\exists x \in I$ s.t. $S(\text{Five Guys}, x)$ T truth set $= \{\text{burgers}, \text{cola}\}$

$\forall x \in R, \; S(x, \text{espresso})$ F truth set $= \{S, D, McD\}$

$\exists x \in I$ s.t. $\sim S(\text{McDonald's}, x)$ F truth set $= \{\ \}$

$\forall x \in I, \; F(x) \leftrightarrow \sim B(x)$

$\exists x \in I$ s.t. $F(x) \wedge S(\text{Starbucks}, x)$

$\forall x \in R, \; S(x, \text{burgers}) \rightarrow S(x, \text{cola})$

Multiple Quantifiers

	Dunkin'	Starbucks	McDonald's	Five Guys	$F(x)$	$B(x)$
donuts	✓	✓	✓		✓	
hash browns	✓		✓		✓	
coffee	✓	✓	✓			✓
espresso	✓	✓	✓			✓
burgers			✓	✓	✓	
cola	✓		✓	✓		✓

$\forall x \in R, \underbrace{\exists y \in I \text{ s.t. } B(y) \wedge S(x,y)}_{\text{there is a beverage that } x \text{ sells}}$ T

work outside in: $\forall x \in R$, $P(x)$ is T exactly when every $x \in R$ makes $P(x)$ T

so check them all

$P(x)$ here is "$\exists y \in I$ s.t. $B(y) \wedge S(x,y)$"

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Dunkin}, y)$ T?

that is, is there a $y \in I$ that makes $B(y) \wedge S(\text{Dunkin'}, y)$ T?

the truth set for

yes, any of {coffee, espresso, cola}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Starbucks}, y)$ T? y truth set {coffee, espresso}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{McDonald's}, y)$ T? y truth set {coffee, espresso, cola}

is $\exists y \in I$ s.t. $B(y) \wedge S(\text{Five Guys}, y)$ T? y

truth set {cola}

all $x \in R$ make $P(x)$ T, so $\forall x \in R, P(x)$ is T

$\forall x \in I, \exists y \in I$ s.t. $(F(x) \wedge \sim B(y)) \vee (\sim F(x) \wedge B(y))$

Something is sold at every store

$\forall x \in R, \exists y \in I$ s.t. $S(x,y)$

or

$\exists y \in I$ s.t. $\forall x \in R, S(x,y)$??