Statements

statement: a sentence that is either true or false

Avelo flies to New Haven. T

(and x y) Southwest flies to White Plains. F

The airline is bankrupt. not a statement

what airline??

compound statement: built from simpler statements using conjunctions like and, or

Avelo flies to Bradley and Spirit flies to BWI

United flies to Providence or American flies to New Haven

$\wedge$ and

$\vee$ or

statement form: replace simple statements w/ variables, conjunctions with symbols

a = " AA flies to AUS "

T b = " XP flies to BWI "

o = " UA flies to OAK "

T s = " UA flies to SFO "

F t = " B6 flies to TUL "

y = " UA flies to YHM "

AA flies to AUS or UA flies to SFO

$a \vee s$

XP flies to BWI and AA flies to AUS

$b \wedge a$

Either B6 flies to TUL, and XP flies to BWI, or UA flies to SFO

$(t \wedge b) \vee s$ T

B6 flies to TUL and either XP flies to BWI or UA flies to SFO

$t \wedge (b \vee s)$ F

precedence: $\sim$ $\wedge$ $\vee$

UA does not fly to OAK $\sim o$ $\neg o$

$\sim a \vee b$ equiv to $(\sim a) \vee b$

UA flies to SFO but not to OAK. (but = and)

$s \wedge \sim o$

UA flies to neither OAK nor YHM.

$\sim o \wedge \sim y$

Let x = # students born in January

$1 < x \leq 8$

p = " $1 < x$ "

q = " $x \leq 8$ "

$p \wedge q$

\wedge $\wedge$ and

\vee $\vee$ or

\wedge $\wedge$ and
\vee $\vee$ or
\neg $\sim$ $\neg$ not

$p \wedge q$

Truth Tables

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

p	~p (¬p)
T	F
F	T

inclusive or

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

p	q	r	p∧q	(p∧q)∨r
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Either (B6 flies to TUL and XP flies to BWI) or UA flies to SFO

p	q	r	p∨r	q∨r	(p∨r)∧(q∨r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	F	F	F

Logical Equivalence

Two statement forms being logically equivalent means
for each combo of truth values of variables truth value of stmt forms is same

(same truth table)

commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$	
associative	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \wedge q \wedge r$	$p \vee (q \vee r) \equiv (p \vee q) \vee r$ $p \vee q \vee r$	$x+y+z$ $(x+y)+z$ $x+(y+z)$
distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$x \cdot (y+z) = xy + xz$
identity	$p \wedge t \equiv p$	$p \vee c \equiv p$	$x+0=x$ $x \cdot 1 = x$
negation	$p \wedge \sim p \equiv c$	$p \vee \sim p \equiv t$	t = tautology (always T)
double negative	$\sim\sim p \equiv p$		c = contradiction (always F)
idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$	
universal bound	$p \wedge c \equiv c$	$p \vee t \equiv t$	$x + \infty = \infty$
De Morgan	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$	
absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$	
negation of t,c	$\sim t \equiv c$	$\sim c \equiv t$	

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$$\sim a \vee b \quad ((a \wedge b) \vee (a \wedge \sim b)) \vee (\sim a \wedge \sim b) \equiv (a \wedge (b \vee \sim b)) \vee (\sim a \wedge \sim b) \quad \text{dist}$$
$$\equiv (a \wedge t) \vee (\sim a \wedge \sim b) \quad \text{neg}$$
$$\equiv a \vee (\sim a \wedge \sim b) \quad \text{ident}$$
$$\equiv (a \vee \sim a) \wedge (a \vee \sim b) \quad \text{dist}$$
$$\equiv t \wedge (a \vee \sim b) \quad \text{neg}$$
$$\equiv a \vee \sim b \quad \text{ident}$$

It is not the case that both UA flies to SFO (p) and UA flies to OAK (q).

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

Either UA doesn't fly to SFO or UA doesn't fly to OAK.

It is not the case that either UA flies to OAK or UA flies to YHM.

Either UA doesn't fly to SFO or UA doesn't fly to OAK.

It is not the case that either UA flies to OAK or UA flies to YHM.

UA flies to neither OAK nor YHM.

$\sim(x \vee y) \equiv \sim x \wedge \sim y$

$\sim(p \vee q) \equiv \underbrace{\sim p}_{\sim(y<3)} \wedge \underbrace{\sim q}_{\sim(z==10)}$

$\sim(y<3)$ z!=10

```
if ( x > 6 && !( y < 3 || z == 10 ))
{
    :
}
else
{
    :
}
```

and not (p or q)

← x > 6 and $y \geq 3$ and $z \neq 10$

```
while ( x > 0 || flag == true )
{
    :
}
```

← $\sim(x>0 \vee \text{flag})$

$\sim(x>0) \wedge \sim\text{flag}$

$x \leq 0 \wedge \sim\text{flag}$

TT -> Statement Forms

Exclusive or: one or the other but not both

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

$(p \wedge \sim q) \vee (\sim p \wedge q)$

$(p \vee q) \wedge \sim (p \wedge q)$

p	q	r	s	t	
T	F	F	T	T	T

$p \wedge \sim q \wedge \sim r \wedge s \wedge t$

Conditionals

If $\underbrace{\text{CPSC 474 has more than 100 students}}_{p}$, then $\underbrace{\text{CPSC 474 has a TF}}_{q}$.

T F

If UVa is the 2023 national champion in basketball, then there is no final exam in CPSC 202.

\rightarrow

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$$p \to q \equiv \sim p \vee q$$